CAMOSUN

Name: ______________________________

Mark: $\overline{}$ **25**

MATH 251 (Winter, 2026)

Test 3

1. (4 marks) Find all the cube roots of i. Write your answer in both Euler form and rectangular form, using exact values (no decimals).

2. (7 marks) Let

$$A = \begin{bmatrix} 1 & -5 & 3 & -3 & -3 \\ -4 & -2 & 4 & -2 & -5 \\ -1 & 3 & 5 & -1 & -2 \\ -4 & 4 & 4 & 1 & 5 \\ 3 & -3 & -3 & 0 & 5 \end{bmatrix}, \quad \mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 4 & 0 & 0 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Verify that $\mathbf{x}_1$ is an eigenvector of A and find its associated eigenvalue, λ_1.

(b) Given that $\lambda_2 = -6$ is an eigenvalue of A, and B is the RREF of $A+6I$, find an eigenvector, $\mathbf{x}_2$, associated with λ_2.

(c) Given that $\lambda_3 = 9$ and $\lambda_4 = 1$ are two additional eigenvalues of A, use the trace of A to find the 5^{th} eigenvalue, λ_5, of A.

(d) Using the eigenvalues of A, compute $\det(A)$.

(e) Is A invertible? Briefly explain.

3. (4 marks) Use Cramer's Rule to solve for x in the following system of equations.

$$\begin{cases} 2x - 3y \phantom{{}+5z} = 5 \\ x + 6y + 5z = -5 \\ 2x + 3y + z = 8 \end{cases}$$

4. (5 marks) Suppose A is a 2×2 matrix such that

$$A\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\3\end{bmatrix} \qquad \text{and} \qquad A\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix}.$$

(a) Find a diagonal matrix D and an invertible matrix P so that $P^{-1}AP = D$.

(b) Find A^5.

5. (5 marks) Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ k \\ 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{u} = \begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix},$$

where k is a real number, and suppose that $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ forms an orthogonal basis of $\mathbb{R}^3$.

(a) Find k.

(b) Find the coordinate vector $[\mathbf{u}]_{\mathcal{B}}$ of $\mathbf{u}$ with respect to $\mathcal{B}$.