CAMOSUN

Name: ______________________________

Mark: $\overline{}$ **25**

MATH 251 (Winter, 2026)

Test 2

1. (3 marks) Evaluate $\begin{bmatrix} -5 & 7 \\ 1 & -2 \end{bmatrix}^T - 2\begin{bmatrix} 6 & 9 \\ 7 & -2 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \end{bmatrix}\begin{bmatrix} 4 & 5 \end{bmatrix} + 4I$.

2. (3 marks) For what value(s) of k (if any) does the product of the matrices $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} 4 & 6 \\ 9 & 1 \end{bmatrix}$ commute?

3. (4 marks) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Use the Gauss-Jordan method to find A^{-1}, and then express both A^{-1} and A as products of elementary matrices.

4. (4 marks) Use the LU method to solve the system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}.$$

5. (5 marks) Let $A = \begin{bmatrix} 1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for row(A).

(b) Find a basis for col(A).

(c) Find a basis for null(A).

(d) Find rank(A) and nullity(A).

6. (6 marks)

(a) Complete the definition of what it means for $T : \mathbb{R}^n \to \mathbb{R}^m$ to be a **linear** transformation:

$T(\mathbf{u} + \mathbf{v}) =$ __________________________ for all $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, and

$T(c\mathbf{u}) =$ __________________________ for all $\mathbf{u}$ in $\mathbb{R}^n$ and for all scalars c.

(b) Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4x - 5y \\ 3x - 4y \end{bmatrix}.$$

(i) State the standard matrix for T.

(ii) Find the standard matrix for the inverse transformation T^{-1}.

(c) Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2y + 3 \\ -x + 2 \end{bmatrix}.$$

(i) Evaluate $T(\mathbf{i})$, $T(\mathbf{j})$ and $T(\mathbf{i} + \mathbf{j})$.

(ii) Is T a linear transformation? Briefly explain.