CAMOSUN

Name: SOLUTIONS

Mark: $\overline{}$ **25**

MATH 251 (Winter, 2026)

Test 1

1. (6 marks) Let $\mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ -6 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$.

   (a) Evaluate $\mathbf{u} \times \mathbf{v}$.

   (b) Find the area of the parallelogram formed by $\mathbf{u}$ and $\mathbf{v}$.

   (c) Find the equation of the plane that passes through the origin and that contains $\mathbf{u}$ and $\mathbf{v}$. Write your answer in both (i) vector form and (ii) general form.

a) $\vec{u} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & -6 \\ 1 & 3 & 4 \end{vmatrix} = \begin{vmatrix} -2 & -6 \\ 3 & 4 \end{vmatrix}\hat{\imath} - \begin{vmatrix} 1 & -6 \\ 1 & 4 \end{vmatrix}\hat{\jmath} + \begin{vmatrix} 1 & -2 \\ 1 & 3 \end{vmatrix}\hat{k}$

$= 10\hat{\imath} - 10\hat{\jmath} + 5\hat{k} = \begin{bmatrix} 10 \\ -10 \\ 5 \end{bmatrix}$ or $5\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

b) Area $= \|\vec{u} \times \vec{v}\| = 5\sqrt{2^2 + (-2)^2 + 1^2} = 5 \cdot 3 = 15$

c) (i) $\vec{x} = s\begin{bmatrix} 1 \\ -2 \\ -6 \end{bmatrix} + t\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

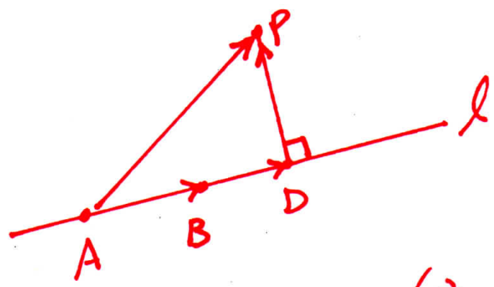
$\vec{u}$ and $\vec{v}$ are direction vectors and $\vec{p} = \vec{0}$ corresponds to point (origin) on plane

(ii) $\vec{n} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ (direction of $\vec{u} \times \vec{v}$)

$\therefore\ 2x - 2y + z = 0$

since plane passes through origin

2. (5 marks) Using projections, find the distance between point $P = (18, -2, 8)$ and the line that passes through the points $A = (1, -1, 4)$ and $B = (2, -3, 6)$.



$\vec{AP} = \begin{bmatrix} 17 \\ -1 \\ 4 \end{bmatrix}$ and $\vec{AB} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

$$\vec{AD} = \text{proj}_{\vec{AB}}(\vec{AP}) = \left(\frac{\vec{AP} \cdot \vec{AB}}{\vec{AB} \cdot \vec{AB}}\right)\vec{AB} = \frac{27}{9}\vec{AB} = 3\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \\ 6 \end{bmatrix}$$

$$\vec{DP} = \vec{AP} - \vec{AD} = \begin{bmatrix} 17 \\ -1 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ -6 \\ 6 \end{bmatrix} = \begin{bmatrix} 14 \\ 5 \\ -2 \end{bmatrix}$$

$$\therefore \text{distance} = \|\vec{DP}\| = \sqrt{14^2 + 5^2 + (-2)^2} = 15$$

3. (5 marks) Use the Gauss-Jordan Elimination method to solve the system of equations.

$$\begin{cases} x_1 - x_2 - 4x_3 = -5 \\ x_2 + 4x_3 = 9 \\ 2x_1 - 2x_2 - 5x_3 = -4 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -4 & -5 \\ 0 & 1 & 4 & 9 \\ 2 & -2 & -5 & -4 \end{array}\right] \rightarrow \begin{array}{c} \\ \\ R_3-2R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & -4 & -5 \\ 0 & 1 & 4 & 9 \\ 0 & 0 & 3 & 6 \end{array}\right]$$

$$\rightarrow \begin{array}{c} R_1+R_2 \\ \\ \frac{1}{3}R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 4 & 9 \\ 0 & 0 & 1 & 2 \end{array}\right] \rightarrow \begin{array}{c} \\ R_2-4R_3 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\therefore\ X_1 = 4,\ X_2 = 1,\ X_3 = 2$

4. (5 marks) Show that the vectors

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}$$

are linearly dependent and find a dependent relationship among them.

Solve $c_1\vec{u} + c_2\vec{v} + c_3\vec{w} = \vec{0}$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & 5 & 2 & 0 \\ 2 & 4 & -2 & 0 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|c} 1 & 0 & -9 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$c_3 = t$ (free)

$c_1 = 9t$, $c_2 = -4t$, $t \in \mathbb{R}$

There are nontrivial solutions, $\therefore$ Vectors are L.D.

For eg. if $t = 1$, $c_1 = 9$, $c_2 = -4$, $c_3 = 1$

and $9\vec{u} - 4\vec{v} + \vec{w} = \vec{0}$.

5. (4 marks) A Valentine's day package of M&M's contains a mixture of red, pink and white M&M's. There are 50 M&M's altogether, there are six more red M&M's than the combined total number of pink and white M&M's, and the number of pink M&M's is seven more than twice the number of white M&M's. Let r, p and w represent the number of red, pink and white M&M's, respectively. Set up a system of linear equations to solve for the number of M&M's of each colour and then construct the augmented matrix associated with your system. **Do not solve the system.**

$r+p+w=50$

$r=p+w+6$

$p=2w+7$

$\Rightarrow$

$r+p+w=50$

$r-p-w=6$

$p-2w=7$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 1 & -1 & -1 & 6 \\ 0 & 1 & -2 & 7 \end{array}\right]$$

(Ans: $r=28$, $p=17$, $w=5$)