Elementary Matrices

Below are descriptions of the forms of each of the three types of elementary matrices. In each case, unless indicated otherwise, all diagonal entries are 1 and non-diagonal entries are 0, as in the identity matrix.

$\mathbf{R_i \leftrightarrow R_j}$: An elementary matrix that swaps row i with row j has the form

$$E = \begin{bmatrix} 1 & & & & & & \\ & \ddots & & & & & \\ & & 0 & & 1 & & \\ & & & \ddots & & & \\ & & 1 & & 0 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{bmatrix} \text{ and its inverse is } E^{-1} = \begin{bmatrix} 1 & & & & & & \\ & \ddots & & & & & \\ & & 0 & & 1 & & \\ & & & \ddots & & & \\ & & 1 & & 0 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{bmatrix}$$

Note: $E^{-1} = E$.

$\mathbf{cR_i}$: An elementary matrix that multiplies row i by a nonzero scalar c has the form

$$E = \begin{bmatrix} 1 & & & & & & \\ & \ddots & & & & & \\ & & 1 & & & & \\ & & & c & & & \\ & & & & 1 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{bmatrix} \text{ and its inverse is } E^{-1} = \begin{bmatrix} 1 & & & & & & \\ & \ddots & & & & & \\ & & 1 & & & & \\ & & & \frac{1}{c} & & & \\ & & & & 1 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{bmatrix}$$

$\mathbf{R_i + cR_j}$: An elementary matrix that adds the nonzero scalar multiple c of row j to row i has the form

$$E = \begin{bmatrix} 1 & & & & & & \\ & \ddots & & & & & \\ & & 1 & & & & \\ & & & \ddots & & & \\ & & c & & 1 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{bmatrix} \text{ and its inverse is } E^{-1} = \begin{bmatrix} 1 & & & & & & \\ & \ddots & & & & & \\ & & 1 & & & & \\ & & & \ddots & & & \\ & & -c & & 1 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{bmatrix}$$

Note: In this matrix E, c would be above and to the right of the main diagonal if $j > i$.