



Name: _____

Mark:
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MATH 220 (Fall, 2024)
Test 1

1. Let $\mathbf{u} = \langle 2, -3, 6 \rangle$ and $\mathbf{v} = \langle 1, 2, -2 \rangle$. Find each of the following.

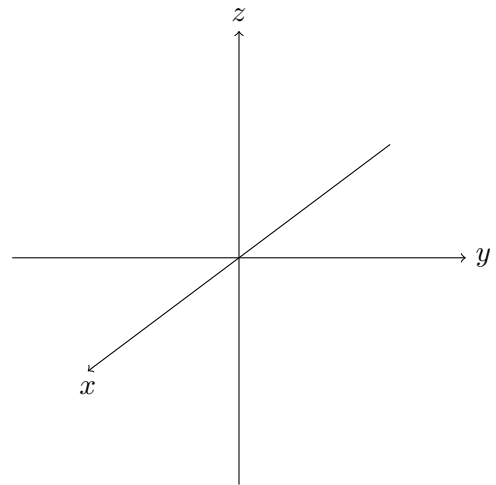
(a) (1 mark) $\mathbf{u} \times \mathbf{v}$

(b) (2 marks) the angle between \mathbf{u} and \mathbf{v} , rounded to the nearest degree

2. (2 marks) Prove that if $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ are vectors in \mathbb{R}^2 and c is a scalar, then $(c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v})$.

3. (3 marks) Find the distance between the point $Q = (1, 2, 3)$ and the plane $2x - 4y + z = 3$.

4. (3 marks) Classify and sketch the quadric surface $9x^2 - 18x + 36y^2 + 4z^2 + 16z = 11$. Write the equation in standard form.



5. Convert the equation $r^2 + z^2 = 9$ in cylindrical coordinates to each of the following.
- (a) (1 mark) rectangular coordinates

(b) (1 mark) spherical coordinates.

6. (4 marks) Consider the curve represented by $\mathbf{r}(t) = \cos(2t)\mathbf{i} + \cos(2t)\mathbf{j} + \sqrt{2}\sin(2t)\mathbf{k}$. Find $\mathbf{r}(s)$ by reparameterizing the curve with respect to arc length. [Recall $s = \int_0^t \|\mathbf{r}'(u)\| du$.]

7. (4 marks) Find $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ for $\mathbf{r}(t) = 4t\mathbf{i} + 3t\mathbf{j} + t^2\mathbf{k}$.

8. (2 marks) Let $\mathbf{T}(\phi) = \langle \cos \phi, \sin \phi \rangle$, where $\phi = \phi(s)$. Show that $\kappa = \left| \frac{d\phi}{ds} \right|$. [Recall $\kappa = \left\| \frac{d\mathbf{T}}{ds} \right\|$.]

9. (2 marks) Show that the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 2y^2}{x^2 + y^2}$ does not exist.