Triple Integrals

Properties of Triple Integrals

Let f and g be continuous over a bounded, solid region Q, and let c be a constant.

1. $\displaystyle\iiint_Q cf(x,y,z)\,dV = c\iiint_Q f(x,y,z)\,dV$

2. $\displaystyle\iiint_Q [f(x,y,z) \pm g(x,y,z)]\,dV = \iiint_Q f(x,y,z)\,dV \pm \iiint_Q g(x,y,z)\,dV$

3. $\displaystyle\iiint_Q f(x,y,z)\,dV \geq 0, \text{ if } f(x,y,z) \geq 0$

4. $\displaystyle\iiint_Q f(x,y,z)\,dV \geq \iiint_Q g(x,y,z)\,dV, \text{ if } f(x,y,z) \geq g(x,y,z)$

5. $\displaystyle\iiint_Q f(x,y,z)\,dV = \iiint_{Q_1} f(x,y,z)\,dV + \iiint_{Q_2} f(x,y,z)\,dV$, where Q is the union of two nonoverlapping solid subregions Q_1 and Q_2.

Fubini's Theorem

Let f be continuous on a solid region Q.

1. If Q is defined by $a \leq x \leq b$, $h_1(x) \leq y \leq h_2(x)$ and $g_1(x,y) \leq z \leq g_2(x,y)$, where h_1, h_2, g_1 and g_2 are continuous functions, then

$$\iiint_Q f(x,y,z)\,dV = \int_a^b \int_{h_1(x)}^{h_2(x)} \int_{g_1(x,y)}^{g_2(x,y)} f(x,y,z)\,dzdydx.$$

2. Similar versions of iterated integral formulas exist for each of the other 5 orders, $dxdydz$, $dxdzdy$, $dydxdz$, $dydzdx$ and $dzdxdy$, and similar corresponding solid regions Q.