Limits and Continuity

Definition of a Limit of a Function of One Variable:
Let f be a function defined on an open interval containing c (except possibly at c), and let L be a real number. Then

$$\lim_{x \to c} f(x) = L$$

if for each $\epsilon > 0$ there exists a $\delta > 0$ such that

$$|f(x) - L| < \epsilon \qquad \text{whenever} \qquad 0 < |x - c| < \delta.$$

Definition of a Limit of a Function of Two Variables:
Let f be a function defined on an open disk centered at (x_0, y_0) (except possibly at (x_0, y_0)), and let L be a real number. Then

$$\lim_{(x,y) \to (x_0,y_0)} f(x, y) = L$$

if for each $\epsilon > 0$ there exists a $\delta > 0$ such that

$$|f(x, y) - L| < \epsilon \qquad \text{whenever} \qquad 0 < \sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta.$$

Definition of a Limit of a Function of Three Variables:
Let f be a function defined on an open sphere centered at (x_0, y_0, z_0) (except possibly at (x_0, y_0, z_0)), and let L be a real number. Then

$$\lim_{(x,y,z) \to (x_0,y_0,z_0)} f(x, y, z) = L$$

if for each $\epsilon > 0$ there exists a $\delta > 0$ such that

$$|f(x, y, z) - L| < \epsilon \qquad \text{whenever} \qquad 0 < \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} < \delta.$$

Definition of Continuity of a Function of One Variable:
A function f of one variable is **continuous at a point $\boldsymbol{c}$** in an open interval I if

$$\lim_{x \to c} f(x) = f(c).$$

The function f is **continuous on the open interval $\boldsymbol{I}$** if it is continuous at every point in I.

Definition of Continuity of a Function of Two Variables:
A function f of two variables is **continuous at a point $\boldsymbol{(x_0, y_0)}$** in an open region R if

$$\lim_{(x,y) \to (x_0,y_0)} f(x, y) = f(x_0, y_0).$$

The function f is **continuous in the open region $\boldsymbol{R}$** if it is continuous at every point in R.

Definition of Continuity of a Function of Three Variables:
A function f of three variables is **continuous at a point $\boldsymbol{(x_0, y_0, z_0)}$** in an open region R if

$$\lim_{(x,y,z) \to (x_0,y_0,z_0)} f(x, y, z) = f(x_0, y_0, z_0).$$

The function f is **continuous in the open region $\boldsymbol{R}$** if it is continuous at every point in R.