Jacobian for Rectangular-to-Spherical Coordinates

If

$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases}$$

then

$$\frac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial \theta} \\ \frac{\partial z}{\partial \rho} & \frac{\partial z}{\partial \phi} & \frac{\partial z}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \sin\phi\cos\theta & \rho\cos\phi\cos\theta & -\rho\sin\phi\sin\theta \\ \sin\phi\sin\theta & \rho\cos\phi\sin\theta & \rho\sin\phi\cos\theta \\ \cos\phi & -\rho\sin\phi & 0 \end{vmatrix}$$

$$= \sin\phi\cos\theta \begin{vmatrix} \rho\cos\phi\sin\theta & \rho\sin\phi\cos\theta \\ -\rho\sin\phi & 0 \end{vmatrix} - \rho\cos\phi\cos\theta \begin{vmatrix} \sin\phi\sin\theta & \rho\sin\phi\cos\theta \\ \cos\phi & 0 \end{vmatrix}$$

$$+ (-\rho\sin\phi\sin\theta) \begin{vmatrix} \sin\phi\sin\theta & \rho\cos\phi\sin\theta \\ \cos\phi & -\rho\sin\phi \end{vmatrix} \quad \textit{(expanding determinant across 1st row)}$$

$$= \sin\phi\cos\theta(\rho^2\sin^2\phi\cos\theta) - \rho\cos\phi\cos\theta(-\rho\cos\phi\sin\phi\cos\theta)$$

$$- \rho\sin\phi\sin\theta(-\rho\sin^2\phi\sin\theta - \rho\cos^2\phi\sin\theta)$$

$$= \rho^2\sin^3\phi\cos^2\theta + \rho^2\cos^2\phi\sin\phi\cos^2\theta + \rho^2\sin^3\phi\sin^2\theta + \rho^2\sin\phi\cos^2\phi\sin^2\theta$$

$$= \rho^2\sin\phi(\sin^2\phi\cos^2\theta + \cos^2\phi\cos^2\theta + \sin^2\phi\sin^2\theta + \cos^2\phi\sin^2\theta)$$

$$= \rho^2\sin\phi(\sin^2\phi + \cos^2\phi)(\sin^2\theta + \cos^2\theta)$$

$$= \rho^2\sin\phi(1)(1)$$

$$= \rho^2\sin\phi.$$