Double Integrals

Properties of Double Integrals

Let f and g be continuous over a closed, bounded plane region R, and let c be a constant.

1. $\displaystyle\iint_R cf(x,y)\,dA = c\iint_R f(x,y)\,dA$

2. $\displaystyle\iint_R [f(x,y) \pm g(x,y)]\,dA = \iint_R f(x,y)\,dA \pm \iint_R g(x,y)\,dA$

3. $\displaystyle\iint_R f(x,y)\,dA \geq 0$, if $f(x,y) \geq 0$

4. $\displaystyle\iint_R f(x,y)\,dA \geq \iint_R g(x,y)\,dA$, if $f(x,y) \geq g(x,y)$

5. $\displaystyle\iint_R f(x,y)\,dA = \iint_{R_1} f(x,y)\,dA + \iint_{R_2} f(x,y)\,dA$, where R is the union of two nonoverlapping subregions R_1 and R_2.

Fubini's Theorem

Let f be continuous on a plane region R.

1. If R is defined by $a \leq x \leq b$ and $g_1(x) \leq y \leq g_2(x)$, where g_1 and g_2 are continuous on $[a,b]$, then
$$\iint_R f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dydx.$$

2. If R is defined by $c \leq y \leq d$ and $h_1(y) \leq x \leq h_2(y)$, where h_1 and h_2 are continuous on $[c,d]$, then
$$\iint_R f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dxdy.$$