Divergence and Stokes's Theorems

Divergence Theorem

Let Q be a solid region bounded by a closed surface S oriented by a unit normal vector directed outward from Q. If $\mathbf{F}$ is a vector field whose component functions have continuous first partial derivatives in Q, then

$$\iint_S \mathbf{F} \cdot \mathbf{N} \, dS = \iiint_Q \boldsymbol{\nabla} \cdot \mathbf{F} \, dV.$$

Stokes's Theorem

Let S be an oriented surface with unit normal vector $\mathbf{N}$, bounded by a piecewise smooth simple closed curve C with a positive orientation. If $\mathbf{F}$ is a vector field whose component functions have continuous first partial derivatives on an open region containing S and C, then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\boldsymbol{\nabla} \times \mathbf{F}) \cdot \mathbf{N} \, dS.$$