

# Derivative Properties of Vector-Valued Functions

**Theorem:** Let  $\mathbf{r}$  and  $\mathbf{u}$  be differentiable vector-valued functions of  $t$ , let  $f$  be a differentiable real-valued function of  $t$ , and let  $c$  be a scalar. Then

(a)  $\frac{d}{dt}[\mathbf{c}\mathbf{r}(t)] = \mathbf{c}\mathbf{r}'(t)$

(b)  $\frac{d}{dt}[\mathbf{r}(t) \pm \mathbf{u}(t)] = \mathbf{r}'(t) \pm \mathbf{u}'(t)$

(c)  $\frac{d}{dt}[f(t)\mathbf{r}(t)] = f(t)\mathbf{r}'(t) + f'(t)\mathbf{r}(t)$

(d)  $\frac{d}{dt}[\mathbf{r}(t) \cdot \mathbf{u}(t)] = \mathbf{r}(t) \cdot \mathbf{u}'(t) + \mathbf{r}'(t) \cdot \mathbf{u}(t)$

(e)  $\frac{d}{dt}[\mathbf{r}(t) \times \mathbf{u}(t)] = \mathbf{r}(t) \times \mathbf{u}'(t) + \mathbf{r}'(t) \times \mathbf{u}(t)$

(f)  $\frac{d}{dt}[\mathbf{r}(f(t))] = \mathbf{r}'(f(t))f'(t)$

(g) If  $\mathbf{r}(t) \cdot \mathbf{r}(t) = c$ , then  $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 0$