Centers of Mass and Moments of Inertia Formulas in Three Dimensions

Mass: $m = \iiint\limits_Q \rho(x,y,z)\,dV$

Moment of mass about the yz-plane: $M_{yz} = \iiint\limits_Q x\rho(x,y,z)\,dV$

Moment of mass about the xz-plane: $M_{xz} = \iiint\limits_Q y\rho(x,y,z)\,dV$

Moment of mass about the xy-plane: $M_{xy} = \iiint\limits_Q z\rho(x,y,z)\,dV$

Center of mass: $(\overline{x},\overline{y},\overline{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m}\right)$

Moment of inertia about the x-axis: $I_x = \iiint\limits_Q (y^2+z^2)\rho(x,y,z)\,dV$

Moment of inertia about the y-axis: $I_y = \iiint\limits_Q (x^2+z^2)\rho(x,y,z)\,dV$

Moment of inertia about the z-axis: $I_z = \iiint\limits_Q (x^2+y^2)\rho(x,y,z)\,dV$

Note that $I_x = I_{xz} + I_{xy}$, $I_y = I_{yz} + I_{xy}$ and $I_z = I_{yz} + I_{xz}$, where

$I_{xy} = \iiint\limits_Q z^2\rho(x,y,z)\,dV$, $I_{xz} = \iiint\limits_Q y^2\rho(x,y,z)\,dV$ and $I_{yz} = \iiint\limits_Q x^2\rho(x,y,z)\,dV$