Centers of Mass and Moments of Inertia Formulas in Two Dimensions

Mass: $m = \iint_R \rho(x,y)\,dA$

Moment of mass about the x-axis: $M_x = \iint_R y\rho(x,y)\,dA$

Moment of mass about the y-axis: $M_y = \iint_R x\rho(x,y)\,dA$

Center of mass: $(\overline{x},\overline{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$

Moment of inertia about the x-axis: $I_x = \iint_R y^2\rho(x,y)\,dA$

Moment of inertia about the y-axis: $I_y = \iint_R x^2\rho(x,y)\,dA$

Moment of inertia about the z-axis: $I_0 = \iint_R (x^2+y^2)\rho(x,y)\,dA = I_x + I_y$

(also called the polar moment of inertia)

Radius of gyration about the x-axis: $\overline{\overline{y}} = \sqrt{\frac{I_x}{m}}$

Radius of gyration about the y-axis: $\overline{\overline{x}} = \sqrt{\frac{I_y}{m}}$