CAMOSUN

Name: SOLUTIONS

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MATH 101 (Fall, 2026)

Test 2

1. (4 marks) Use partial fractions to integrate $\int \frac{x+1}{x^2-6x+9}\,dx$. $= \int \frac{(x+1)}{(x-3)^2}\,dx$

$\frac{x+1}{(x-3)^2} = \frac{A}{x-3} + \frac{B}{(x-3)^2} \Rightarrow x+1 = A(x-3)+B$

$x=3 \Rightarrow B=4$

$x=0 \Rightarrow 1 = -3A+B \Rightarrow 1 = -3A+4 \Rightarrow A=1$

$\therefore \int \left(\frac{1}{x-3} + \frac{4}{(x-3)^2}\right)dx = \ln|x-3| - \frac{4}{x-3} + C$

2. (3 marks) Integrate $\int \sec 2x \tan^3 2x\,dx$.

$= \int \tan^2 2x \sec 2x \tan 2x\,dx = \frac{1}{2}\int (\sec^2 2x - 1)(2\sec 2x \tan 2x)\,dx$

$= \frac{1}{2}\int (u^2-1)\,du$, where $u = \sec 2x$ and $du = 2\sec 2x \tan 2x\,dx$

$= \frac{1}{2}\left[\frac{1}{3}u^3 - u\right] + C = \frac{1}{6}\sec^3 2x - \frac{1}{2}\sec 2x + C$

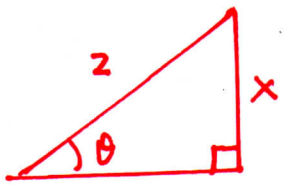
3. (5 marks) Evaluate the definite integral $\int_0^{\sqrt{2}} \sqrt{4-x^2}\,dx$.

Let $x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$\sin\theta = \frac{x}{2}$

$\theta = \arcsin\frac{x}{2}$



$\sqrt{4-x^2} = 2\cos\theta$

$x = 0 \Rightarrow \theta = \arcsin 0 = 0$

$x = \sqrt{2} \Rightarrow \theta = \arcsin\frac{\sqrt{2}}{2} = \frac{\pi}{4}$

$$\int_0^{\sqrt{2}} \sqrt{4-x^2}\,dx = \int_0^{\frac{\pi}{4}} 2\cos\theta \cdot 2\cos\theta\, d\theta = 4\int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$$

$$= 4\int_0^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2}\, d\theta = 4\left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{4}}$$

$$= \left[2\theta + \sin 2\theta\right]_0^{\frac{\pi}{4}} = \frac{\pi}{2} + 1$$

4. (3 marks) Integrate by parts by letting $u = 1 - \ln x$ and simplify your answer.

$$\int \frac{1 - \ln x}{x^2}\, dx$$

$u = 1 - \ln x \qquad dv = \frac{1}{x^2}\, dx$

$du = -\frac{1}{x}\, dx \qquad v = -\frac{1}{x}$

$$\int \frac{1-\ln x}{x^2}\, dx = (1-\ln x)\left(-\tfrac{1}{x}\right) - \int \left(-\tfrac{1}{x}\right)\left(-\tfrac{1}{x}\right) dx$$

$$= -\frac{1}{x} + \frac{\ln x}{x} - \int \frac{1}{x^2}\, dx = -\frac{1}{x} + \frac{\ln x}{x} + \frac{1}{x} + C$$

$$= \frac{\ln x}{x} + C$$

5. (3 marks) Evaluate the limit $\lim_{x \to 0^+} (\sin x + \cos x)^{1/x}$. Form 1^∞

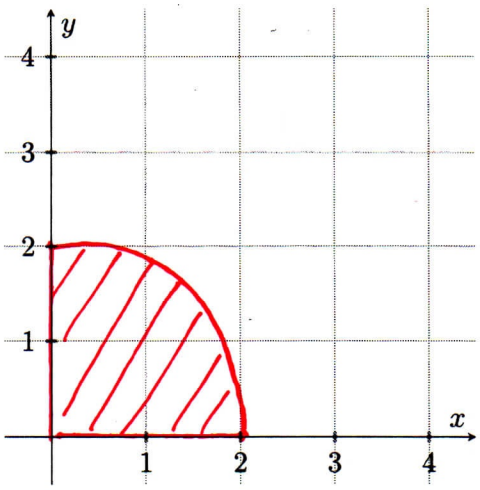
Let $L = \lim_{x \to 0^+} (\sin x + \cos x)^{\frac{1}{x}}$

Then $\ln L = \lim_{x \to 0^+} \ln(\sin x + \cos x)^{\frac{1}{x}} = \lim_{x \to 0^+} \frac{\ln(\sin x + \cos x)}{x}$ Form $\frac{0}{0}$

L'HR $= \lim_{x \to 0^+} \frac{\frac{\cos x - \sin x}{\sin x + \cos x}}{1} = 1$ $\quad \therefore L = e$

6. Consider the region in the first quadrant bounded by the curves $y = \sqrt{4 - x^2}$, $x = 0$ and $y = 0$.

(a) (2 marks) Sketch and shade the region.



(b) (1 mark) Find the area of the region by using a geometry formula (not integration).

$A = \frac{1}{4}\pi(2)^2 = \pi$ (area of $\frac{1}{4}$-circle of radius $r=2$)

(c) (4 marks) Using your answer from part (b), find the y-coordinate, $\overline{y}$, of the centroid of the region.

$\Delta A_x = \frac{\sqrt{4-x^2}}{2} \cdot \sqrt{4-x^2}\,\Delta x = \frac{4-x^2}{2}\Delta x = \left(2 - \frac{1}{2}x^2\right)\Delta x$

for $0 \le x \le 2$

$A_x = \int_0^2 \left(2 - \frac{1}{2}x^2\right)dx = \left[2x - \frac{1}{6}x^3\right]_0^2 = \frac{8}{3}$

$\therefore \overline{y} = \frac{A_x}{A} = \frac{8}{3\pi} \approx 0.8488$