CAMOSUN

Name: SOLUTIONS

Mark: ___/25

MATH 101 (Fall, 2026)

Test 1

1. (3 marks) Evaluate the definite integral $\int_4^6 \frac{dx}{\sqrt{-x^2+8x}}$.

$$= \int_4^6 \frac{dx}{\sqrt{-(x^2-8x)}} = \int_4^6 \frac{dx}{\sqrt{16-(x^2-8x+16)}} = \int_4^6 \frac{dx}{\sqrt{16-(x-4)^2}} = \arcsin\left(\frac{x-4}{4}\right)\Big|_4^6$$

$$= \arcsin\frac{1}{2} - \arcsin 0 = \frac{\pi}{6} - 0 = \frac{\pi}{6}$$

2. (3 marks) Differentiate $f(x) = \operatorname{arcsec}(\cosh x)$, assuming $x > 0$, and simplify your answer.

$$f'(x) = \frac{\sinh x}{|\cosh x|\sqrt{\cosh^2 x - 1}} = \frac{\sinh x}{\cosh x \sqrt{\sinh^2 x}} = \frac{\sinh x}{(\cosh x)(\sinh x)} = \frac{1}{\cosh x} = \operatorname{sech} x$$

$\cosh x > 0$

$\sinh x > 0$ for $x > 0$

3. (3 marks) By first converting hyperbolic functions to exponential form, simplify the expression

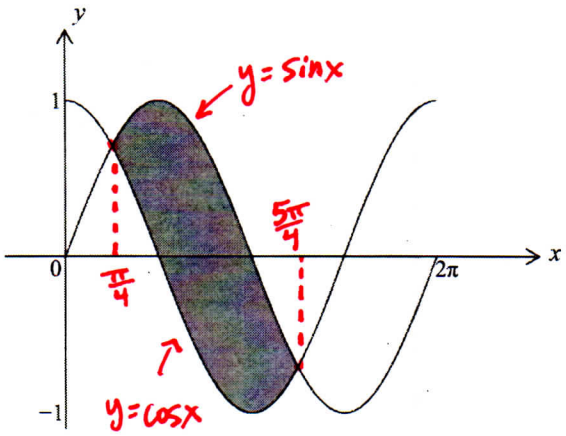
$$\frac{\sinh(\ln x) + \cosh(\ln x)}{\ln(\sinh x + \cosh x)}.$$

$$= \frac{\frac{e^{\ln x} - e^{-\ln x}}{2} + \frac{e^{\ln x} + e^{-\ln x}}{2}}{\ln\left(\frac{e^x - e^{-x}}{2} + \frac{e^x + e^{-x}}{2}\right)} = \frac{e^{\ln x}}{\ln(e^x)} = \frac{x}{x} = 1$$

4. (4 marks) Find the area of the shaded region between the curves $y = \sin x$ and $y = \cos x$.

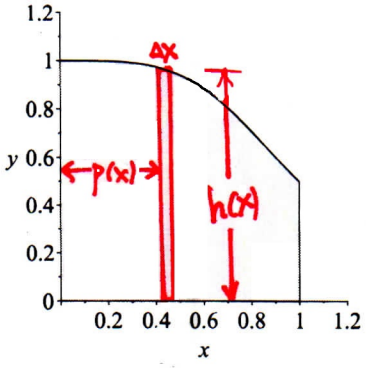
Intersection: $\sin x = \cos x$

$\Rightarrow \frac{\sin x}{\cos x} = 1 \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$



$$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx = \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

5. (4 marks) Use the shell method to find the volume of the solid formed by revolving the region bounded by the curves $y = \dfrac{1}{1+x^4}$ and $y = 0$ for $0 \le x \le 1$ (as illustrated) about the y-axis.



$\Delta V = 2\pi p(x)\, h(x)\, \Delta x$ for $0 \le x \le 1$

$= 2\pi x \cdot \dfrac{1}{1+x^4}\, \Delta x$

$V = 2\pi \int_0^1 \dfrac{x}{1+x^4}\, dx = \pi \int_0^1 \dfrac{1}{1+(x^2)^2}\, (2x)\, dx$

$= \pi \int_0^1 \dfrac{du}{1+u^2}$, where $u = x^2$, $du = 2x\,dx$

$x = 0 \Rightarrow u = 0$ and $x = 1 \Rightarrow u = 1$

$= \pi \arctan u \Big|_0^1 = \pi\left(\dfrac{\pi}{4} - 0\right) = \dfrac{\pi^2}{4}$

6. (2 marks) Find the arc length of the curve $y = -\dfrac{3}{4}x + 2$ for $0 \le x \le 4$.

$S = \int_0^4 \sqrt{1 + \left[\frac{dy}{dx}\right]^2}\, dx = \int_0^4 \sqrt{1 + \left(-\frac{3}{4}\right)^2}\, dx = \int_0^4 \frac{5}{4}\, dx = \frac{5}{4}x \Big|_0^4 = 5$

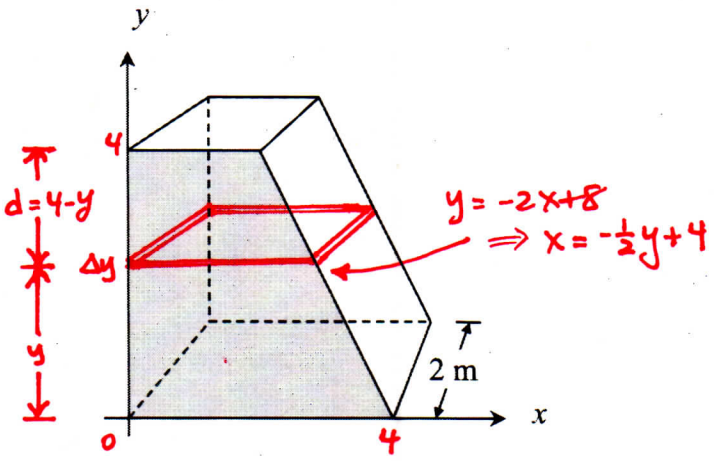
7. (6 marks) Each end of a tank is in the shape of the trapezoidal region (shown shaded in the illustration below) bounded by the lines $y = 4$, $y = -2x + 8$, and the coordinate axes, where x and y are measured in meters. The length of the tank is 2 meters and the bottom of the tank is sitting on the ground. If the tank is full of brine, which weighs 12,000 N/m^3, then how much work is required to empty the tank by pumping all the brine up and out through a hole at the top of the tank?

$\Delta V = 2(-\frac{1}{2}y+4)\Delta y = (-y+8)\Delta y$ for $0 \le y \le 4$

$\Delta F = 12000(-y+8)\Delta y$

$d = 4-y$

$\Delta W = \Delta F \cdot d = 12000(-y+8)(4-y)\Delta y$



$W = 12000\int_0^4 (y^2-12y+32)\,dy = 12000\left[\frac{1}{3}y^3 - 6y^2 + 32y\right]_0^4$

$= 12000\left(\frac{160}{3}\right) = 640\,000$ J (or 640 kJ)