CAMOSUN

Name: SOLUTIONS

Mark: ___/25

MATH 100 (Fall, 2026)

Test 1B

1. Consider the following equation relating x and y.

$$x(x+y^2) = 1 - y^2(1-x)$$

(a) (2 marks) Solve for y in terms of x.

$x^2+xy^2 = 1 - y^2 + xy^2$

$y^2 = 1-x^2$ ← unit circle $x^2+y^2=1$

$y = \pm\sqrt{1-x^2}$

(b) (1 mark) Does the equation define y as a function of x? Briefly explain.

No since some x-values (i.e. if $-1<x<1$) yield 2 y-values (e.g. if $x=0$, then $y=\pm 1$)

2. (2 marks) Solve the equation for θ. Find **all** solutions.

$$1 - \underbrace{2\sin\theta\cos\theta}_{\sin 2\theta} = 0$$

$\sin 2\theta = 1$

$2\theta = \frac{\pi}{2} + 2n\pi, \quad n \in \mathbb{Z}$

$\theta = \frac{\pi}{4} + n\pi, \quad n \in \mathbb{Z}$

3. (3 marks) Evaluate the limit $\lim_{x\to 9} \dfrac{\sqrt{18-x}-\sqrt{x}}{9-x}$.

$$= \lim_{x\to 9} \frac{\sqrt{18-x}-\sqrt{x}}{9-x} \cdot \frac{\sqrt{18-x}+\sqrt{x}}{\sqrt{18-x}+\sqrt{x}} = \lim_{x\to 9} \frac{18-x-x}{(9-x)(\sqrt{18-x}+\sqrt{x})}$$

$$= \lim_{x\to 9} \frac{18-2x}{(9-x)(\sqrt{18-x}+\sqrt{x})} = \lim_{x\to 9} \frac{2(9-x)}{(9-x)(\sqrt{18-x}+\sqrt{x})} = \lim_{x\to 9} \frac{2}{\sqrt{18-x}+\sqrt{x}} = \frac{2}{3+3} = \frac{1}{3}$$

4. (3 marks) Let $f(x) = x^4 - 8x + 9$ on the interval $[1, 2]$ and let $k = 7$. Verify that the conditions of the Intermediate Value Theorem are satisfied, guaranteeing the existence of at least one number c in the given interval for which $f(c) = k$.

f is Continuous on the closed interval $[1,2]$

$f(1) = 2 < 7$ and $f(2) = 9 > 7$ (i.e. $k=7$ is between $f(1)$ and $f(2)$).

$\therefore$ IVT Conditions satisfied.

5. Let f and g be differentiable functions and define $h(x) = f(x)g(x)$.

(a) (1 mark) Write the product rule formula for $h'(x)$ in terms of f, g and their derivatives.

$h'(x) =$ $f(x)g'(x) + f'(x)g(x)$

(b) (1 mark) Suppose the values of f and g and their derivatives are defined at various values of x as given in the following chart. Evaluate $h'(2)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	3
2	3	1	2	5
3	1	3	3	2

$h'(2) = f(2)g'(2) + f'(2)g(2)$

$= 3 \cdot 5 + 2 \cdot 1 = 15 + 2 = 17$

6. (a) (1 mark) State the limit definition of a derivative of $f(x)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

(b) (3 marks) Use the **limit definition** of a derivative to find $f'(x)$, where $f(x) = \dfrac{x}{x+1}$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{x+\Delta x}{x+\Delta x+1} - \frac{x}{x+1}}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)(x+1) - x(x+\Delta x+1)}{(x+\Delta x+1)(x+1)\,\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + x + \Delta x x + \Delta x - x^2 - x\Delta x - x}{(x+\Delta x+1)(x+1)\,\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x}{(x+\Delta x+1)(x+1)\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{(x+\Delta x+1)(x+1)} = \frac{1}{(x+1)^2}$$

7. (3 marks) Find the equation of the tangent line to the following curve at $x = 9$. Express your answer in slope-intercept form.

$$y = 2\sqrt{x} + \frac{9}{x} \qquad y' = \frac{1}{\sqrt{x}} - \frac{9}{x^2}$$

At $x=9$, $y=7$ and $y' = \frac{2}{9}$ (slope)

$\therefore\ y - 7 = \frac{2}{9}(x-9)$

$y = \frac{2}{9}x + 5$

8. Differentiate the following functions and simplify your answer.

(a) (1 mark) $y = x^5 + 3x^2 - 4x - 2$

$y' = 5x^4 + 6x - 4$

(b) (2 marks) $f(\theta) = 4\sec^3(5\theta - 1)$

$f'(\theta) = 12\sec^2(5\theta-1)\sec(5\theta-1)\tan(5\theta-1)\cdot 5$

$= 60\sec^3(5\theta-1)\tan(5\theta-1)$

(c) (2 marks) $y = \dfrac{4x-7}{3x+2}$

$y' = \dfrac{(3x+2)(4) - (4x-7)(3)}{(3x+2)^2} = \dfrac{12x+8-12x+21}{(3x+2)^2} = \dfrac{29}{(3x+2)^2}$