CAMOSUN

Name: SOLUTIONS

Mark: ___/25

MATH 100 (Fall, 2026)

Test 1A

1. Consider the following equation relating x and y.

$$y(y+x^2) = 1 - x^2(1-y)$$

(a) (2 marks) Solve for y in terms of x.

$y^2 + x^2y = 1 - x^2 + x^2y$

$y^2 = 1 - x^2$ ← unit circle $x^2+y^2=1$

$y = \pm\sqrt{1-x^2}$

(b) (1 mark) Does the equation define y as a function of x? Briefly explain.

No since some x-values (i.e. if $-1<x<1$) yield 2 y-values (e.g., if $x=0$, then $y=\pm 1$)

2. (2 marks) Solve the equation for θ. Find **all** solutions.

$$2\sin\theta\cos\theta + 1 = 0$$

$\sin 2\theta = -1$

$2\theta = \frac{3\pi}{2} + 2n\pi, \quad n \in \mathbb{Z}$

$\theta = \frac{3\pi}{4} + n\pi, \quad n \in \mathbb{Z}$

3. (3 marks) Evaluate the limit $\lim_{x\to 4} \dfrac{\sqrt{8-x}-\sqrt{x}}{4-x}$.

$$= \lim_{x\to 4} \frac{\sqrt{8-x}-\sqrt{x}}{4-x} \cdot \frac{\sqrt{8-x}+\sqrt{x}}{\sqrt{8-x}+\sqrt{x}} = \lim_{x\to 4} \frac{8-x-x}{(4-x)(\sqrt{8-x}+\sqrt{x})} = \lim_{x\to 4} \frac{8-2x}{(4-x)(\sqrt{8-x}+\sqrt{x})}$$

$$= \lim_{x\to 4} \frac{2(4-x)}{(4-x)(\sqrt{8-x}+\sqrt{x})} = \lim_{x\to 4} \frac{2}{\sqrt{8-x}+\sqrt{x}} = \frac{2}{2+2} = \frac{1}{2}$$

4. (3 marks) Let $f(x) = x^3 - 4x + 5$ on the interval $[1, 2]$ and let $k = 3$. Verify that the conditions of the Intermediate Value Theorem are satisfied, guaranteeing the existence of at least one number c in the given interval for which $f(c) = k$.

f is continuous on the closed interval $[1,2]$

$f(1) = 2 < 3$ and $f(2) = 5 > 3$ (i.e. $k=3$ is between $f(1)$ and $f(2)$).

$\therefore$ IVT conditions satisfied.

5. Let f and g be differentiable functions and define $h(x) = f(g(x))$.

(a) (1 mark) Write the chain rule formula for $h'(x)$ in terms of f, g and their derivatives.

$h'(x) = f'(g(x))\, g'(x)$

(b) (1 mark) Suppose the values of f and g and their derivatives are defined at various values of x as given in the following chart. Evaluate $h'(2)$.

$h'(2) = f'(g(2))\, g'(2) = f'(1) \cdot 5 = 4 \cdot 5 = 20$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	3
2	3	1	2	5
3	1	3	3	2

6. (a) (1 mark) State the limit definition of a derivative of $f(x)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

(b) (3 marks) Use the **limit definition** of a derivative to find $f'(x)$, where $f(x) = \dfrac{x}{x-1}$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{x+\Delta x}{x+\Delta x - 1} - \frac{x}{x-1}}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)(x-1) - x(x+\Delta x - 1)}{(x+\Delta x - 1)(x-1)\,\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 - x + \Delta x\, x - \Delta x - x^2 - x\,\Delta x + x}{(x+\Delta x - 1)(x-1)\,\Delta x} = \lim_{\Delta x \to 0} \frac{-\Delta x}{(x+\Delta x - 1)(x-1)\,\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x - 1)(x-1)} = \frac{-1}{(x-1)^2}$$

7. (3 marks) Find the equation of the tangent line to the following curve at $x = 4$. Express your answer in slope-intercept form.

$$y = 6\sqrt{x} + \frac{16}{x}$$

$$y' = \frac{3}{\sqrt{x}} - \frac{16}{x^2}$$

At $x = 4$, $y = 16$ and $y' = \frac{1}{2}$ (slope)

$$\therefore \quad y - 16 = \tfrac{1}{2}(x-4)$$

$$y = \tfrac{1}{2}x + 14$$

8. Differentiate the following functions and simplify your answer.

(a) (1 mark) $y = x^4 - 7x^2 + 5x - 1$

$y' = 4x^3 - 14x + 5$

(b) (2 marks) $f(\theta) = 3\sec^3(7\theta + 2)$

$f'(\theta) = 9\sec^2(7\theta+2) \cdot \sec(7\theta+2)\tan(7\theta+2) \cdot 7$

$= 63\sec^3(7\theta+2)\tan(7\theta+2)$

(c) (2 marks) $y = \dfrac{5x+3}{2x-7}$

$y' = \dfrac{(2x-7)(5) - (5x+3)(2)}{(2x-7)^2} = \dfrac{10x-35-10x-6}{(2x-7)^2} = \dfrac{-41}{(2x-7)^2}$