CAMOSUN

Name: SOLUTIONS

Mark: ___/25

MATH 100
Assignment 2

1. (1 mark) Evaluate the limit. If it does not exist, then determine whether it is ∞ or $-\infty$ or neither.

$$\lim_{x\to 2\pi^+} \frac{x}{\sin(x-\pi)} = \frac{2\pi}{\sin(\pi^+)} = \frac{2\pi}{0^-} = -\infty$$

(d.n.e.)

2. (2 marks) Let $f(x) = \dfrac{x^2-9}{x^3-x^2-6x}$.

(a) Find the equations of all vertical asymptotes (if any) of f.

$$f(x) = \frac{(x-3)(x+3)}{x(x-3)(x+2)} = \frac{x+3}{x(x+2)} \quad \text{for } x \neq 0, 3, -2$$

$\therefore$ $x=0$ and $x=-2$ are V.A.

(b) Find the limit of $f(x)$ at all removable discontinuities (if any) of f.

$x=3$ is a removable discontinuity

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x+3}{x(x+2)} = \frac{6}{15} = \frac{2}{5}$$

3. (3 marks) Use the **limit definition** of a derivative to find $f'(x)$, where $f(x) = \dfrac{2}{x^2+4}$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{2}{(x+\Delta x)^2+4} - \frac{2}{x^2+4}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2(x^2+4) - 2\left[(x+\Delta x)^2+4\right]}{\left[(x+\Delta x)^2+4\right](x^2+4)\Delta x} = \lim_{\Delta x \to 0} \frac{2x^2+8-2(x^2+2x\Delta x+(\Delta x)^2)-8}{\left[(x+\Delta x)^2+4\right](x^2+4)\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x^2-2x^2-4x\Delta x-2(\Delta x)^2}{\left[(x+\Delta x)^2+4\right](x^2+4)\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x(-4x-2\Delta x)}{\left[(x+\Delta x)^2+4\right](x^2+4)\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-4x-2\Delta x}{\left[(x+\Delta x)^2+4\right](x^2+4)} = \frac{-4x}{(x^2+4)(x^2+4)} = \frac{-4x}{(x^2+4)^2}$$

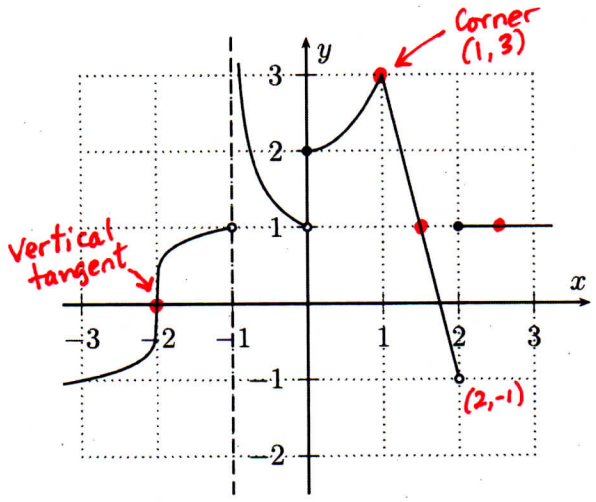
4. (2 marks) Use the alternative form of the derivative, $f'(c) = \lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c}$, to find $f'(3)$ for the function $f(x) = \sqrt{2x+19}$.

$$f'(3) = \lim_{x \to 3} \frac{f(x) - f(3)}{x-3} = \lim_{x \to 3} \frac{\sqrt{2x+19}-5}{x-3} \cdot \frac{\sqrt{2x+19}+5}{\sqrt{2x+19}+5}$$

$$= \lim_{x \to 3} \frac{(2x+19)-25}{(x-3)(\sqrt{2x+19}+5)} = \lim_{x \to 3} \frac{2x-6}{(x-3)(\sqrt{2x+19}+5)}$$

$$= \lim_{x \to 3} \frac{2}{\sqrt{2x+19}+5} = \frac{2}{10} = \frac{1}{5}$$

5. (2 marks) Consider the graph of the function $f(x)$ given below.



(a) List all of the x-values at which f is **not** continuous. $-1, 0, 2$

(b) List all of the x-values at which f is **not** differentiable. $-2, -1, 0, 1, 2$

(c) Evaluate $f'(5/2)$ and $f'(3/2)$ by relating these derivative values to the slope of the graph.

$f'(5/2) = 0$

$f'(3/2) = \frac{3-(-1)}{1-2} = \frac{4}{-1} = -4$

6. (2 marks) Differentiate by using basic derivative rules, but **not** the product or quotient rules. Simplify your answer by expressing it in the form of a single fraction using only positive exponents (i.e. in a form similar to how y itself is written).

$$y = \frac{7x^{3/2} - 12x + 2}{6\sqrt{x}} = \frac{7}{6}x - 2x^{1/2} + \frac{1}{3}x^{-1/2}$$

$$y' = \frac{7}{6} - x^{-1/2} - \frac{1}{6}x^{-3/2} = \frac{7}{6} - \frac{1}{x^{1/2}} - \frac{1}{6x^{3/2}} = \frac{7x^{3/2} - 6x - 1}{6x^{3/2}}$$

7. (2 marks) Find the derivative of $g(t) = \dfrac{4t^2 - t + 6}{2t^2 + 3}$.

$$g'(t) = \frac{(2t^2+3)(8t-1) - (4t^2-t+6)(4t)}{(2t^2+3)^2}$$

$$= \frac{16t^3 - 2t^2 + 24t - 3 - 16t^3 + 4t^2 - 24t}{(2t^2+3)^2}$$

$$= \frac{2t^2 - 3}{(2t^2+3)^2}$$

8. (3 marks) Find the equation of the tangent line to the curve $y = 8x\tan x$ at $x = \pi/4$. Write your answer in slope-intercept form.

At $x = \pi/4$, $y = 8\left(\frac{\pi}{4}\right)\tan\frac{\pi}{4} = 2\pi$ $\therefore$ point is $\left(\frac{\pi}{4}, 2\pi\right)$

$y' = 8x\sec^2 x + 8\tan x$

At $x = \frac{\pi}{4}$, $y' = 8\left(\frac{\pi}{4}\right)\sec^2\frac{\pi}{4} + 8\tan\frac{\pi}{4} = 4\pi + 8$ (slope)

$\therefore\ y - 2\pi = (4\pi+8)\left(x - \frac{\pi}{4}\right)$

$y = (4\pi+8)x - \pi^2 - 2\pi + 2\pi$

$y = (4\pi+8)x - \pi^2$ $\quad (y \approx 20.57x - 9.87)$

9. (2 marks) The position function (measured in meters) of a particle is given by

$$s(t) = 2t^3 - 9t^2 - 50t,$$

for $t \geq 0$, where t is measured in seconds. When does the particle reach a velocity of 10 m/s?

$s'(t) = 6t^2 - 18t - 50 = 10$

$\Rightarrow 6t^2 - 18t - 60 = 0$

$6(t^2 - 3t - 10) = 0$

$6(t-5)(t+2) = 0$

$t = 5$ or $t = -2$ (discard)

$\therefore$ Particle's velocity is 10 m/s after 5 seconds.

10. (1 mark) Let $f(x) = \cos x$. Find $f^{(100)}(x)$, the 100^{th} derivative of $f(x)$.

$f'(x) = -\sin x$, $f''(x) = -\cos x$, $f'''(x) = \sin x$, $f^{(4)}(x) = \cos x$, ...

The derivatives cycle through the sequence $-\sin x, -\cos x, \sin x, \cos x$, whereby every 4^{th} derivative (i.e. $4^{\text{th}}, 8^{\text{th}}, 12^{\text{th}}, \ldots, 100^{\text{th}}$) is $\cos x$.

$\therefore f^{(100)}(x) = \cos x$

(since 100 is divisible by 4)

11. (2 marks) Find the equation of the tangent line to the curve $y = \sqrt{x^2 + 22x - 7}$ at $x = 1$.

At $x=1$, $y=4$. $\therefore$ point is $(1,4)$

$$y' = \frac{2x+22}{2\sqrt{x^2+22x-7}} = \frac{x+11}{\sqrt{x^2+22x-7}}$$

At $x=1$, $y' = \frac{12}{4} = 3$ (slope)

$\therefore y - 4 = 3(x-1)$

$y = 3x + 1$

12. Differentiate the following functions. Simplify your answers.

(a) (1 mark) $y = 5\sec^2(7\theta + 4)$

$$y' = 10\sec(7\theta+4)\cdot \sec(7\theta+4)\tan(7\theta+4)\cdot 7$$

$$= 70\sec^2(7\theta+4)\tan(7\theta+4)$$

(b) (2 marks) $f(x) = \dfrac{x^2 - 9}{(6 - x^2)^{2/3}}$

$$f'(x) = \frac{(6-x^2)^{2/3}(2x) - (x^2-9)\left(\frac{2}{3}\right)(6-x^2)^{-1/3}(-2x)}{\left[(6-x^2)^{2/3}\right]^2}$$

$$= \frac{\frac{2}{3}x(6-x^2)^{-1/3}\left[3(6-x^2)+2(x^2-9)\right]}{(6-x^2)^{4/3}}$$

$$= \frac{2x\,(18-3x^2+2x^2-18)}{3(6-x^2)^{5/3}}$$

$$= \frac{2x\,(-x^2)}{3(6-x^2)^{5/3}}$$

$$= \frac{-2x^3}{3(6-x^2)^{5/3}}$$