CAMOSUN

Name: ______________________

MATH 251 (Winter, 2019)
Term Test 3
by George Ballinger

Answer the questions in the space provided.
This test has 5 questions for a total of 25 marks.

1. (4 marks) Evaluate $(\sqrt{3}+i)^{11}$. Express your answer in both polar and rectangular form, using exact values in both cases.

2. (4 marks) Use Cramer's Rule to find the y-value in the solution of the following system of linear equations. You do not need to solve for the other variables.

$$\begin{cases} 2x \qquad\quad - 6z = 10 \\ x + 4y - 2z = -8 \\ 5x + 2y + z = 3 \end{cases}$$

3. (5 marks) Let $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$.

   (a) Find the eigenvalues and corresponding eigenspaces of A.

   (b) If possible, find a diagonal matrix D and an invertible matrix P so that $P^{-1}AP = D$. If it's not possible, then explain why.

4. (5 marks) Consider the following matrix.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & x \end{bmatrix}$$

For what value(s) of x (if any) is A *not* diagonalizable? Justify.

5. (7 marks) Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}, \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 5 \\ -5 \\ 9 \end{bmatrix}$$

and define $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and $W = \operatorname{span}(\mathbf{v}_1, \mathbf{v}_2)$.

(a) Show that $\mathcal{B}$ is an orthogonal basis for $\mathbb{R}^3$.

(b) Express $\mathbf{v}$ as a linear combination of the vectors in $\mathcal{B}$. In other words, find $[\mathbf{v}]_{\mathcal{B}}$.

(c) Find a basis for $W^{\perp}$.

(d) Find $\operatorname{proj}_W(\mathbf{v})$ and $\operatorname{perp}_W(\mathbf{v})$.

(e) Construct an orthonormal basis for $\mathbb{R}^3$ using the vectors from $\mathcal{B}$.

Note: If you need more space, you may continue your answer on the back of this page.