CAMOSUN

Name: ______________________________

Mark: $\overline{}$ **25**

MATH 251 (Winter, 2022)

Test 3

1. (5 marks) Find all complex number solutions of the equation

$$z^3 + 8i = 0.$$

Express your answer(s) in the rectangular form $a + bi$ using exact values.

2. (6 marks) Let $A = \begin{bmatrix} 3 & -2 & -8 \\ 2 & 6 & 1 \\ -1 & 0 & 2 \end{bmatrix}$.

   (a) Calculate $\operatorname{tr}(A)$, the trace of matrix A.

   (b) Use the cofactor method to find A^{-1}.

   (c) Find the volume of the parallelepiped determined by the column vectors of A.

3. (6 marks) Diagonalize the matrix

$$A = \begin{bmatrix} 1 & -4 \\ -1 & -2 \end{bmatrix}$$

by factoring it into a product $A = PDP^{-1}$, where P is an invertible matrix and D is a diagonal matrix.

4. (8 marks) Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

and define $W = \text{span}(\mathbf{v}_1, \mathbf{v}_2)$.

(a) Show that $\{\mathbf{v}_1, \mathbf{v}_2\}$ is an orthogonal basis for W.

(b) Find an orthonormal basis for W.

(c) Find $\text{proj}_W(\mathbf{v})$ and $\text{perp}_W(\mathbf{v})$.

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(d) Find a basis for $W^{\perp}$.

(e) Is your basis from part (d) orthogonal? Briefly explain.