CAMOSUN

Name: ____________________

MATH 251 (Winter, 2019)
Term Test 2
by George Ballinger

Answer the questions in the space provided. This test has 6 questions for a total of 25 marks.

1. (3 marks) Find the matrix A that satisfies the following.

$$(I_2 + 3A^T)^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

2. (3 marks) Solve the given matrix equation for X. Assume that all matrices are invertible. Simplify your answer.

$$BX^{-1} = (A - I)(A^{-1} + I) + A^{-1}$$

3. (5 marks) Consider the following system of linear equations.

$$\begin{cases} x - \ \ y - 3z = -2 \\ x - 2y - 5z = 3 \\ -x + \ \ y + 4z = 10 \end{cases}$$

(a) Express the linear system in the matrix form: $A\mathbf{x} = \mathbf{b}$.

(b) Find A^{-1} using the Gauss-Jordan method for computing the inverse.

(c) Use A^{-1} to solve the system.

4. (4 marks) Find an LU-factorization of the following matrix.

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 8 & -6 & -4 \\ -4 & 3 & 1 \end{bmatrix}$$

5. (5 marks) Consider the following matrix A with its RREF.

$$A = \begin{bmatrix} 1 & 4 & -1 & 8 & 13 \\ -3 & 0 & -9 & -7 & -17 \\ 6 & 2 & 16 & 1 & 6 \\ 5 & -1 & 16 & 3 & 12 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 3 & 0 & 1 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find bases for the row space, column space and null space of A.

(b) Find $\text{rank}(A)$ and $\text{nullity}(A)$.

6. (5 marks) Consider the linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$ that rotates a vector counter-clockwise by $60°$ and then reflects it about the line $y = -x$.

   (a) Find the standard matrix of T.

   (b) Use your answer in part (a) to find $T(\mathbf{v})$, where $\mathbf{v} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$. Give an exact answer.