CAMOSUN

Name: ____________________

Mark: $\overline{}$ **25**

MATH 251 (Winter, 2022)

Test 2

1. (4 marks) Factor $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 0 & 1 \\ 0 & 3 & 0 \end{bmatrix}$ into a product of elementary matrices.

2. (3 marks) Solve the system of equations

$$\begin{cases} 11x_1 + 6x_2 - 3x_3 - 7x_4 = 3 \\ -26x_1 - 13x_2 + 8x_3 + 16x_4 = -1 \\ -61x_1 - 31x_2 + 18x_3 + 38x_4 = 0 \\ -x_1 - x_2 + x_4 = 4 \end{cases}$$

using the fact that

$$\begin{bmatrix} 11 & 6 & -3 & -7 \\ -26 & -13 & 8 & 16 \\ -61 & -31 & 18 & 38 \\ -1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 & 4 \\ 4 & -3 & 2 & 0 \\ 1 & 5 & -2 & 3 \\ 6 & 0 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

3. (8 marks) A matrix B is said to be a square root of a matrix A if $B^2 = A$. Consider a 2×2 matrix B having the form

$$B = \begin{bmatrix} 3 & k^2 \\ -k & -3 \end{bmatrix},$$

where k is a real number. Find the value(s) of k (if any) if

(a) B is a square root of the identity matrix I_2.

(b) $B\mathbf{x} = 6\mathbf{x}$, where $\mathbf{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

(c) B is a symmetric matrix.

(d) B is an invertible matrix.

4. (6 marks) Suppose

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 & 2 \\ 2 & 1 & 3 & 2 & 5 \\ 3 & 2 & 5 & 4 & 7 \\ 4 & 2 & 6 & 1 & 7 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 2 \\ 2 & 3 & 5 & 6 \\ 2 & 2 & 4 & 1 \\ 2 & 5 & 7 & 7 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the column space of A **consisting of columns of** A.

(b) Find a basis for the row space of A **consisting of rows of** A.

(c) Find a basis for the null space of A.

(d) Find $\text{rank}(A)$ and $\text{nullity}(A)$.

5. (4 marks) Suppose T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ satisfying

$$T(\mathbf{i}) = \begin{bmatrix} -5 \\ 2 \end{bmatrix} \quad \text{and} \quad T(\mathbf{j}) = \begin{bmatrix} -7 \\ 3 \end{bmatrix}.$$

(a) Evaluate $T(2\mathbf{i} - 3\mathbf{j})$.

(b) Find the standard matrix for T^{-1}.

(c) Evaluate $T^{-1}(\mathbf{i})$ and $T^{-1}(\mathbf{j})$.