CAMOSUN

Name: ____________________

Mark: $\overline{}$ **25**

MATH 251 (Winter, 2022)

Test 1

1. (6 marks) Consider the parallelogram formed by vectors $\mathbf{u} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ -5 \\ 0 \end{bmatrix}$.

   (a) Find the area of the parallelogram. Give an exact, simplified answer.

   (b) Find the angle $0^\circ \leq \theta \leq 180^\circ$ between $\mathbf{u}$ and $\mathbf{v}$. Round your answer to two decimal places.

   (c) Find vectors representing each of the diagonals of the parallelogram.

2. (6 marks) Consider the points $A = (1, 2, 3)$, $B = (3, 5, -3)$ and $C = (11, 10, -6)$.
   (a) Find a unit vector that points in the direction of $\overrightarrow{AB}$.
   (b) Find an equation, in vector form, of the line passing through points A and B.
   (c) Find the distance between point C and the line in part (b).

3. (4 marks) Consider the line defined parametrically by

$$\begin{cases} x = 7 - 2t \\ y = 1 + 5t \\ z = -6 + 3t. \end{cases}$$

(a) Find the coordinates of the point where the line intersects the xy-plane.

(b) Find an equation, in general form, of the plane passing through the point $P = (1, 2, -1)$ that is perpendicular to the given line.

4. (5 marks) Use the Gauss-Jordan Elimination method to find all the solutions of the system of linear equations. Write your answer in column vector form. Clearly show your steps, including your row operations.

$$\begin{cases} \quad\; 2x_2 - 4x_3 - 12x_4 = 14 \\ x_1 \quad\quad + 3x_3 + \;\; 6x_4 = -9 \\ \quad\; 2x_2 - 3x_3 - 11x_4 = 9 \end{cases}$$

5. (2 marks) Set up, **but do not solve**, a homogenous system of linear equations that could be used to balance the following unbalanced chemical equation involving the elements Mn, O, H and Cl.

$$\mathrm{MnO_2} + \mathrm{HCl} \to \mathrm{MnCl_2} + \mathrm{H_2O} + \mathrm{Cl_2}$$

6. (2 marks) Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 4 \\ -2 \\ 3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 5 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 4 \\ 5 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 7 \\ -9 \\ -2 \\ -4 \\ 9 \end{bmatrix}$$

and suppose

$$\begin{bmatrix} 1 & 2 & 0 & 7 \\ 3 & -1 & 4 & -9 \\ 4 & 3 & 5 & -2 \\ -2 & 5 & 2 & -4 \\ 3 & 2 & 1 & 9 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

If possible, express $\mathbf{v}_4$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$. If not possible, explain why.