CAMOSUN

Name: ______________________

Mark: $\overline{}$ **25**

MATH 251 (Winter, 2024)

Test 3

1. (4 marks) Evaluate $4\underline{/120^\circ} - 2\underline{/60^\circ}$. Express your answer in phasor form, $r\underline{/\theta}$, using exact values for both r and θ (i.e. no decimals).

2. (6 marks) Let $A = \begin{bmatrix} 9 & -4 \\ 5 & 1 \end{bmatrix}$.

   (a) Find the area of the parallelogram formed by the column vectors of A.

   (b) Is $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ an eigenvector of A? Briefly explain.

   (c) Find the eigenvalues of A.

3. (5 marks) Let $A = \begin{bmatrix} 1 & 2 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$.

   (a) Find the eigenvalues and corresponding eigenspaces of A.

   (b) If possible, find a diagonal matrix D and an invertible matrix P so that $P^{-1}AP = D$. If it's not possible, then explain why.

4. (4 marks) Use Cramer's Rule to solve for x_2 in the system of linear equations $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 0 & 1 & 5 & 0 \\ 3 & 1 & 0 & 3 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 8 \\ 9 \\ 2 \\ 7 \end{bmatrix}.$$

5. (6 marks) Let

$$\mathbf{v}_1 = \begin{bmatrix} -1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ 1/2 \end{bmatrix}, \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix},$$

and note that $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent. Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ be a basis for the subspace $W = \operatorname{span}(\mathbf{v}_1, \mathbf{v}_2)$ of $\mathbb{R}^4$.

(a) Is $\mathcal{B}$ an orthonormal set? Briefly explain.

(b) What vectors, if any, belong to both W and $W^{\perp}$?

(c) Find $\operatorname{proj}_W(\mathbf{v})$ and $\operatorname{perp}_W(\mathbf{v})$.