CAMOSUN

Name: ______________________

Mark: $\dfrac{\quad}{\mathbf{25}}$

MATH 251 (Winter, 2023)

Test 2

1. (5 marks) Consider matrix $A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 2 \\ 2 & 6 & 1 \end{bmatrix}$.

   (a) Find an LU factorization of A.

   (b) Given that $A^{-1} = \begin{bmatrix} 11 & -3 & -5 \\ -4 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix}$, solve the system of equations $\begin{cases} x_1 + 3x_2 + x_3 = 3 \\ x_2 + 2x_3 = 9 \\ 2x_1 + 6x_2 + x_3 = 1 \end{cases}$.

2. (6 marks) Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.

   (a) Evaluate $A^2 + 2A - 3I$.

   (b) Write A as a product of elementary matrices.

3. (4 marks) Consider the following matrix A with its RREF.

$$A = \begin{bmatrix} 1 & -1 & -7 \\ -2 & 2 & 14 \\ 3 & -1 & -13 \\ -5 & -1 & 11 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the column space of A.

(b) Find a basis for the row space of A.

(c) Find a basis for the null space of A.

4. (4 marks) Consider the 2×8 matrix

$$A = \begin{bmatrix} k & 0 & k & 0 & k & 0 & k & 0 \\ 0 & k & 0 & k & 0 & k & 0 & k \end{bmatrix},$$

where k is a real number.

(a) If $k \neq 0$, then what are the rank and nullity of A?

(b) For what value(s) of k, if any, is $AA^T = I$?

5. (6 marks)

(a) Complete the definition of what it means for $T : \mathbb{R}^n \to \mathbb{R}^m$ to be a **linear** transformation:

$T(\mathbf{u}+\mathbf{v}) =$ ______________________________ for all $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, and

$T(c\mathbf{u}) =$ ______________________________ for all $\mathbf{u}$ in $\mathbb{R}^n$ and for all scalars c.

(b) Given the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined as follows, find a formula for the inverse transformation T^{-1}.

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} y \\ 2x - 3y \end{bmatrix}, \qquad T^{-1}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \qquad\qquad \\ \\ \end{bmatrix}$$

(c) Show, by way of a counterexample to one of the two properties defining a linear transformation, that the transformation $T : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = |x| + |y|,$$

is **not** a linear transformation.