CAMOSUN

Name: ______________________

Mark: $\frac{\quad}{25}$

MATH 251 (Winter, 2024)

Test 2

1. (3 marks) Evaluate $A^{-1} + 2A^T - 4I$, where $A = \begin{bmatrix} -8 & 5 \\ -3 & 2 \end{bmatrix}$.

2. (3 marks) Let S be the set of vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$ such that $|x| = |y| = |z|$.

   (a) Give examples of two linearly independent vectors, $\mathbf{u}$ and $\mathbf{v}$, belonging to S.

   (b) Use your vectors from part (a) to prove that S is not a subspace of $\mathbb{R}^3$.

3. (6 marks) Let $A = \begin{bmatrix} 2 & -3 \\ -6 & 5 \end{bmatrix}$.

   (a) Find an LU factorization of A.

   (b) Use the LU method to solve the system

$$\begin{cases} 2x_1 - 3x_2 = -7 \\ -6x_1 + 5x_2 = 1. \end{cases}$$

4. (4 marks) Let $A = \begin{bmatrix} 0 & a & 0 \\ 1 & 0 & 0 \\ b & 0 & 1 \end{bmatrix}$, where a and b are scalars and $a \neq 0$.

   (a) Use the Gauss-Jordan method to find A^{-1}.

   (b) Find a and b if A is symmetric.

5. (5 marks) Suppose

$$A = \begin{bmatrix} 4 & 8 & 1 & 9 & 5 \\ 3 & 6 & 2 & 8 & 4 \\ 2 & 4 & 3 & 7 & 3 \\ 1 & 2 & 4 & 6 & 3 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find a basis $\mathcal{B}$ for the column space of A consisting of columns of A.

(b) For each column vector $\mathbf{v}$ of A that is *not* in your basis $\mathcal{B}$ from part (a), find its coordinate vector with respect to the basis $\mathcal{B}$, i.e. find $[\mathbf{v}]_{\mathcal{B}}$.

(c) Find $\text{rank}(A)$, $\text{rank}(A^T)$, $\text{nullity}(A)$ and $\text{nullity}(A^T)$.

6. (4 marks) Suppose $S : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation defined by

$$S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4y \\ 2x \end{bmatrix}.$$

(a) What is the standard matrix for S?

(b) Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that rotates a vector 60° counter-clockwise and then performs the transformation S. Find the standard matrix for T. Give an exact, simplified answer (no decimals).