Name: $\qquad$

Mark:
25

## MATH 251 (Winter, 2024) <br> Test 1

1. (6 marks) Using projections, find the distance between point $B=(-3,-4,7)$ and the plane $2 x+y-2 z=3$ and find the coordinates of the point $A$ in the plane that is closest to point $B$.
2. Find the equation of the plane containing the points $A=(1,2,3), B=(2,0,3)$ and $C=(4,1,2)$, and write your answer in
(a) (3 marks) vector form.
(b) (3 marks) general form.
3. (a) (4 marks) Solve the system of linear equations by using the Gauss-Jordan Elimination method. Write your answer in column vector form. Clearly show your steps, including your row operations.

$$
\left\{\begin{aligned}
x+3 y-3 z & =0 \\
x+3 z & =0 \\
2 x-y+8 z & =0
\end{aligned}\right.
$$

(b) (2 marks) Using your answer from part (a) and by referring to the definition of linear dependence, briefly explain why, or why not, the following vectors are linearly dependent.

$$
\mathbf{u}=\left[\begin{array}{l}
1 \\
1 \\
2
\end{array}\right], \mathbf{v}=\left[\begin{array}{r}
3 \\
0 \\
-1
\end{array}\right], \mathbf{w}=\left[\begin{array}{r}
-3 \\
3 \\
8
\end{array}\right]
$$

4. Find the value(s) of $k$ (if any) for which $\left[\begin{array}{l}k \\ 1\end{array}\right]$ and $\left[\begin{array}{c}-3 \\ k+4\end{array}\right]$ are
(a) (2 marks) orthogonal vectors.
(b) (3 marks) distinct parallel vectors.
5. (2 marks) Set up, but do not solve, a homogeneous system of linear equations that could be used to balance the following unbalanced chemical equation involving the elements $\mathrm{C}, \mathrm{O}$ and H.

$$
\mathrm{CO}_{2}+\mathrm{H}_{2} \mathrm{O} \rightarrow \mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}+\mathrm{O}_{2}
$$

