

Name:	

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MATH 251 (Winter, 2024) Test 1

1. (6 marks) Using projections, find the distance between point B = (-3, -4, 7) and the plane 2x + y - 2z = 3 and find the coordinates of the point A in the plane that is closest to point B.

- 2. Find the equation of the plane containing the points A = (1, 2, 3), B = (2, 0, 3) and C = (4, 1, 2), and write your answer in
 - (a) (3 marks) vector form.

(b) (3 marks) general form.

3. (a) (4 marks) Solve the system of linear equations by using the **Gauss-Jordan Elimination** method. Write your answer in column vector form. Clearly show your steps, including your row operations.

$$\begin{cases} x + 3y - 3z = 0 \\ x + 3z = 0 \\ 2x - y + 8z = 0 \end{cases}$$

(b) (2 marks) Using your answer from part (a) and by referring to the definition of linear dependence, briefly explain why, or why not, the following vectors are linearly dependent.

$$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \ \mathbf{v} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, \ \mathbf{w} = \begin{bmatrix} -3 \\ 3 \\ 8 \end{bmatrix}$$

- 4. Find the value(s) of k (if any) for which $\begin{bmatrix} k \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ k+4 \end{bmatrix}$ are
 - (a) (2 marks) orthogonal vectors.

(b) (3 marks) distinct parallel vectors.

5. (2 marks) Set up, **but do not solve**, a *homogeneous* system of linear equations that could be used to balance the following unbalanced chemical equation involving the elements C, O and H.

$$CO_2 + H_2O \rightarrow C_6H_{12}O_6 + O_2$$