



Name: _____

Mark:
25

MATH 251 (Winter, 2023)

Test 1

1. (7 marks) Consider the points $A = (1, 2, 3)$, $B = (2, 1, 5)$ and $C = (3, 2, 1)$.
 - (a) Find $\overrightarrow{AB} + \overrightarrow{AC}$, $\overrightarrow{AB} \cdot \overrightarrow{AC}$, and $\overrightarrow{AB} \times \overrightarrow{AC}$.
 - (b) Find the area of triangle $\triangle ABC$. Give an exact, simplified answer.
 - (c) Find the measure of angle $\angle A$ in triangle $\triangle ABC$ rounded to the nearest degree.

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2. (7 marks) Consider the point $Q = (4, 1, 3)$ and the plane \mathcal{P} given by $x - 2y + 3z = 5$.
- (a) Find the equation of the line, in parametric form, that passes through the point Q and is perpendicular to the plane \mathcal{P} .
 - (b) Find the equation of the plane, in general form, that is parallel to the plane \mathcal{P} and that contains the point Q .
 - (c) Using projections, find the distance between the point Q and the plane \mathcal{P} .

3. (6 marks) Consider the system of linear equations

$$\begin{cases} x_1 + 8x_2 + 2x_3 = 10 \\ x_1 + 3x_2 + 2x_3 = 5 \\ 3x_1 + 4x_2 + 6x_3 = 10 \end{cases}$$

- (a) Use the Gauss-Jordan Elimination Method to find all of the solutions of the system.
- (b) Find all of the solutions of the system such that x_1 , x_2 , and x_3 are nonnegative integers.

4. (5 marks) Let \mathbf{u} , \mathbf{v} , and \mathbf{w} be vectors in \mathbb{R}^n .

(a) Briefly explain what it means for \mathbf{w} to be in $\text{span}(\mathbf{u}, \mathbf{v})$.

(b) Let $\mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 1 \\ -4 \\ 4 \end{bmatrix}$. Determine whether or not \mathbf{w} is in $\text{span}(\mathbf{u}, \mathbf{v})$.