

Symmetric Matrices and Orthogonal Diagonalization

Theorem: If A is an $n \times n$ real symmetric matrix, then

- (a) the eigenvalues of A are real.
- (b) any two eigenvectors corresponding to distinct eigenvalues of A are orthogonal.

Definition: An $n \times n$ matrix A is **orthogonally diagonalizable** if there exists an orthogonal matrix Q and a diagonal matrix D such that $Q^T A Q = D$.

Spectral Theorem: Let A be an $n \times n$ real matrix. Then A is symmetric if and only if it is orthogonally diagonalizable.