

Spanning Sets and Linear Independence of Vectors

Definition: If $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \dots, \mathbf{v}_k\}$ is a set of vectors in \mathbb{R}^n , then the set of all linear combinations of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \dots, \mathbf{v}_k$ is called the **span** of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \dots, \mathbf{v}_k$ and is denoted $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \dots, \mathbf{v}_k)$ or $\text{span}(S)$. If $\text{span}(S) = \mathbb{R}^n$, then S is called a **spanning set** for \mathbb{R}^n .

Definition: A set of vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \dots, \mathbf{v}_k$ in \mathbb{R}^n is **linearly dependent** (LD) if there are scalars $c_1, c_2, c_3, \dots, c_k$, *at least one of which is nonzero*, such that

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + \cdots + c_k\mathbf{v}_k = \mathbf{0}.$$

A set of vectors that is not linearly dependent is called **linearly independent** (LI).

Note: If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \dots, \mathbf{v}_k$ are linearly independent, then $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + \cdots + c_k\mathbf{v}_k = \mathbf{0}$ if and only if $c_1 = c_2 = c_3 = \cdots = c_k = 0$.