Spanning Sets and Linear Independence of Matrices

Definition: If $S = \{A_1, A_2, A_3, \ldots, A_k\}$ is a set of matrices of the same size, then the set of all linear combinations of $A_1, A_2, A_3, \ldots, A_k$ is called the **span** of $A_1, A_2, A_3, \ldots, A_k$ and is denoted $\text{span}(A_1, A_2, A_3, \ldots, A_k)$ or $\text{span}(S)$.

Definition: A set of matrices $A_1, A_2, A_3, \ldots, A_k$ of the same size is **linearly dependent** (LD) if there are scalars $c_1, c_2, c_3, \ldots, c_k$, *at least one of which is nonzero*, such that

$$c_1A_1 + c_2A_2 + c_3A_3 + \cdots + c_kA_k = O.$$

A set of matrices that is not linearly dependent is called **linearly independent** (LI).

Note: If $A_1, A_2, A_3, \ldots, A_k$ are linearly independent, then $c_1A_1+c_2A_2+c_3A_3+\cdots+c_kA_k = O$ if and only if $c_1 = c_2 = c_3 = \cdots = c_k = 0$.