Section Exploration: The Cross Product
Answers to Recommended Problems

Below are answers to the recommended problems from the "Exploration: The Cross Product" section. Vectors have been written in row form for convenience of typing, but they should be in column form.

1. (a) [3,3,-3]

   (b) [-3,-3,3]

   (c) **0**

   (d) [1,-2,1]

2. Here we calculate $\mathbf{e}_1 \times \mathbf{e}_2$ and show it equals $\mathbf{e}_3$ and do the same for the other two cross products. Recall $\mathbf{e}_1=[1,0,0]$, $\mathbf{e}_2=[0,1,0]$, and $\mathbf{e}_3=[0,0,1]$.

4. (a) [3,3,-3]·[x,y,z]=[3,3,-3]·[1,0,-2] is one possible answer; converting to general form and simplifying gives x+y-z=3.

   (b) [-5,5,5]·[x,y,z]=[-5,5,5]·[0,-1,1] is one possible answer; converting to general form and simplifying gives x-y-z=0.

5. (a) If we calculate $\mathbf{v}\times\mathbf{u}$, where $\mathbf{v}=[v_1,v_2,v_3]$ and $\mathbf{u}=[u_1,u_2,u_3]$, using whichever technique we prefer, and factor out a negative sign, we should be left with $\mathbf{u}\times\mathbf{v}$, proving $\mathbf{v}\times\mathbf{u}=-(\mathbf{u}\times\mathbf{v})$.

6. (a) If we let $\mathbf{u}=[u_1,u_2,u_3]$, $\mathbf{v}=[v_1,v_2,v_3]$, and $\mathbf{w}=[w_1,w_2,w_3]$ and work out the cross products $\mathbf{v}\times\mathbf{w}$ and $\mathbf{u}\times\mathbf{v}$, and then later work out and expand the dot products $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})$ and $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}$, we should find that they are equal, each consisting of the same six terms.

8. (c) $\frac{\sqrt{62}}{2}$