Row Echelon Form

Definition: A matrix is in **row echelon form** (REF) if it satisfies the following properties:

(i) Any rows consisting entirely of zeros are at the bottom.

(ii) In each nonzero row, the first nonzero entry (called the **leading entry**) is in a column to the left of any leading entries below it.

Examples: The following matrices are in row echelon form, with leading entries highlighted:

$$\begin{bmatrix} \boxed{4} & -3 & 2 \\ 0 & \boxed{-3} & 1 \\ 0 & 0 & \boxed{-7} \end{bmatrix}, \begin{bmatrix} \boxed{2} & 0 & 1 \\ 0 & \boxed{6} & 4 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \boxed{1} & 3 & 5 & 0 \\ 0 & 0 & 0 & \boxed{4} & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} \boxed{-3} & 5 & -4 & 1 & -2 \\ 0 & \boxed{1} & 3 & 1 & 0 \\ 0 & 0 & 0 & \boxed{5} & 8 \\ 0 & 0 & 0 & 0 & \boxed{2} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Definition: A matrix is in **reduced row echelon form** (RREF) if it satisfies the following properties:

(i) It is in row echelon form (REF).

(ii) The leading entry in each nonzero row is a 1 (called a **leading 1**).

(iii) Each column containing a leading 1 has zeros everywhere else.

Examples: The following matrices are in reduced row echelon form, with leading 1's highlighted:

$$\begin{bmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \end{bmatrix}, \begin{bmatrix} \boxed{1} & 0 & -3 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \boxed{1} & 6 & 0 & 1 \\ 0 & 0 & 0 & \boxed{1} & -7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} \boxed{1} & 0 & 2 & 0 & 0 \\ 0 & \boxed{1} & -8 & 0 & 0 \\ 0 & 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 & \boxed{1} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$