# Properties of Invertible Matrices

**Properties of the Inverse of a Matrix:**

Let $A$ and $B$ be invertible matrices of the same size, $c$ be a non-zero scalar, and $n$ be a nonnegative integer. Then $A^{-1}$, $cA$, $AB$, $A^T$ and $A^n$ are also invertible and

(a) $(A^{-1})^{-1} = A$

(b) $(cA)^{-1} = \frac{1}{c}A^{-1}$

(c) $(AB)^{-1} = B^{-1}A^{-1}$

(d) $(A^T)^{-1} = (A^{-1})^T$

(e) $(A^n)^{-1} = (A^{-1})^n$

**The Fundamental Theorem of Invertible Matrices:**

Let $A$ be an $n \times n$ matrix. The following statements are equivalent:

(a) $A$ is invertible.

(b) $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b}$ in $\mathbb{R}^n$.

(c) $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

(d) The reduced row echelon form (RREF) of $A$ is $I_n$.

(e) $A$ is a product of elementary matrices.

(f) $\text{rank}(A) = n$

(g) $\text{nullity}(A) = 0$

(h) The column vectors of $A$ are linearly independent.

(i) The column vectors of $A$ span $\mathbb{R}^n$.

(j) The column vectors of $A$ form a basis for $\mathbb{R}^n$.

(k) The row vectors of $A$ are linearly independent.

(l) The row vectors of $A$ span $\mathbb{R}^n$.

(m) The row vectors of $A$ form a basis for $\mathbb{R}^n$.

(n) $\det(A) \neq 0$

(o) 0 is not an eigenvalue of $A$.