Appendix C

Exercises C

In Exercises 1–14, evaluate the given expression and write your answer in the form $a + bi$.

1. $(3 + 2i) + (5 - 6i)$

2. $(1 + i) - (2 - 3i)$

3. $(5 + 2i)(3 + i)$

4. $(\frac{1}{2} + i)^2$

5. $\overline{7 + 4i}$

6. $\overline{3i(1 - 2i)}$

7. $\dfrac{1}{1 + i}$

8. $\dfrac{2}{3 - 4i}$

9. $\dfrac{4 - i}{1 + 3i}$

10. $\dfrac{\sqrt{3} + i}{1 + \sqrt{3}i}$

11. i^3

12. i^{2012}

13. $\sqrt{-100}$

14. $\sqrt{-2}\,\sqrt{-18}$

In Exercises 15–18, find the absolute value of the given complex number.

15. $4 + 3i$

16. $1 - i$

17. $1 + 2\sqrt{2}i$

18. $\frac{3}{2} + 2i$

In Exercises 19–22, write the given complex number in polar form using its principal argument.

19. $2 - 2i$

20. $5i$

21. $\sqrt{3} + i$

22. $-3 - 4i$

In Exercises 23–26, find the polar form of zw, z/w, and $1/z$ by first putting z and w in polar form.

23. $z = -1 + i$, $w = \sqrt{3} + i$

24. $z = 1 + \sqrt{3}i$, $w = 2\sqrt{3} - 2i$

25. $z = 4 + 4i$, $w = 2i$

26. $z = 3(\sqrt{3} + i)$, $w = -1 - i$

In Exercises 27–30, find the indicated power using De Moivre's Theorem.

27. $(1 - i)^8$

28. $(\frac{1}{2} + \frac{1}{2}i)^{10}$

29. $(1 + \sqrt{3}i)^5$

30. $(2\sqrt{3} - 2i)^3$

In Exercises 31–34, find the indicated roots and sketch them in the complex plane.

31. The eighth roots of 1

32. The sixth roots of -64

33. The cube roots of i

34. The cube roots of $4\sqrt{2} + 4\sqrt{2}i$

In Exercises 35–38, write the given number in the form $a + bi$.

35. $e^{-i\pi/2}$

36. $2e^{i\pi/3}$

37. $-e^{1-i\pi}$

38. $e^{(1+i\pi)/2}$

39. Prove the following properties of the complex conjugate:
(a) $\overline{\overline{z}} = z$
(b) $\overline{z + w} = \overline{z} + \overline{w}$
(c) $\overline{zw} = \overline{z}\,\overline{w}$
(d) If $z \neq 0$, then $\overline{(w/z)} = \overline{w}/\overline{z}$.
(e) z is real if and only if $\overline{z} = z$.
(f) $z + \overline{z} = 2\text{Re}\, z$ and $z - \overline{z} = 2i\, \text{Im}\, z$.

40. Prove the following properties of absolute value:
(a) $|\overline{z}| = |z|$
(b) $|zw| = |z||w|$
(c) If $z \neq 0$, then $|1/z| = 1/|z|$.
(d) $|z| = 0$ if and only if $z = 0$.
(e) $\text{Re}\, z \leq |z|$
(f) $|z + w| \leq |z| + |w|$ [**Hint:** Square the left hand side and expand using the identity $|z|^2 = z\overline{z}$. Exercises 39(f) and 40(e) are useful.]

41. (a) Derive the double angle formulas $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$ by expanding $(\cos\theta + i\sin\theta)^2$ and comparing the result with the answer given by De Moivre's Theorem.
(b) Imitate the method of part (a) to derive formulas for $\cos 3\theta$ and $\sin 3\theta$.
(c) Imitate the method of part (a) to derive formulas for $\cos 4\theta$ and $\sin 4\theta$.

42. Prove De Moivre's Theorem using mathematical induction.