

MATH 251
MATLAB Assignment

1. (6 marks) Consider the vectors $\mathbf{u} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -5 \\ 8 \\ 9 \end{bmatrix}$.

(a) Write the MATLAB code that creates the column vectors \mathbf{u} and \mathbf{v} .

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>> _____

(b) Write the MATLAB code that calculates the area of the triangle formed by \mathbf{u} , \mathbf{v} and $\mathbf{u} - \mathbf{v}$, and perform the calculation.

>> _____

Answer: _____

(c) Write the MATLAB code that calculates the angle, in degrees, between \mathbf{u} and \mathbf{v} . Perform the calculation and round your answer to the nearest degree.

>> _____

Answer: _____

(d) Write the MATLAB code that calculates $\mathbf{v} - \text{proj}_{\mathbf{u}}(\mathbf{v})$. Perform the calculation and write your answer using fractions.

>> _____

Answer: _____

2. (5 marks) Consider the unbalanced chemical equation for the following redox reaction involving the nine elements Cu, S, C, N, K, I, O, H, and Cl:



To balance the equation one must find positive integers x_1, x_2, \dots, x_8 so that



has the same number of each atom on both sides.

- (a) Set up a homogeneous system of linear equations for the variables x_1, x_2, \dots, x_8 and construct its associated 9×9 augmented matrix.

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- (b) Using MATLAB, find the RREF of the augmented matrix in part (a). Write the result below using fractions, and then, by hand, use the RREF to solve the system of equations and balance the chemical equation.

Answer: $\text{— CuSCN} + \text{— KIO}_3 + \text{— HCl} \longrightarrow \text{— CuSO}_4 + \text{— KCl} + \text{— HCN} + \text{— ICl} + \text{— H}_2\text{O}$

3. (3 marks) Consider the matrix A and the vector \mathbf{b} given by

$$A = \begin{bmatrix} 1 & 5 \\ 1 & 3 \\ 1 & -6 \\ 1 & 1 \\ 1 & -5 \\ 1 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 15 \\ 19 \\ 4 \\ 15 \\ 9 \\ 22 \end{bmatrix}.$$

- (a) Write the MATLAB code that creates A and \mathbf{b} .

>> _____

>> _____

- (b) Write the MATLAB code that calculates $(A^T A)^{-1} A^T \mathbf{b}$. Perform the calculation and write your answer using fractions.

>> _____

Answer: _____

4. (2 marks) Write the MATLAB code to evaluate the following complex number, and then perform the calculation, rounding the real and imaginary parts of your answer to two decimal places.

$$\frac{(0.12 + 0.98i)^{100}}{(0.92 - 0.13i)^{40}}$$

>> _____

Answer: _____

5. (3 marks) Consider the matrix A and the vectors \mathbf{x}_1 , \mathbf{x}_2 , \mathbf{x}_3 given by

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \\ 26 & 27 & 28 & 29 & 30 \end{bmatrix} \quad \text{and} \quad \mathbf{x}_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}.$$

- (a) Using MATLAB, find $\text{rank}(A)$, and then by hand calculate $\text{nullity}(A)$.

Answer: $\text{rank}(A) =$ _____, $\text{nullity}(A) =$ _____

- (b) Using MATLAB, evaluate $A\mathbf{x}_1$, $A\mathbf{x}_2$ and $A\mathbf{x}_3$.

Answer: $A\mathbf{x}_1 =$ _____, $A\mathbf{x}_2 =$ _____, $A\mathbf{x}_3 =$ _____

Which of the vectors \mathbf{x}_1 , \mathbf{x}_2 and \mathbf{x}_3 (if any) are *not* in $\text{null}(A)$?

Answer: _____

Briefly explain: _____

6. (2 marks) Write the MATLAB code that transforms the vector $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$ by applying the linear transformation R to it a total of 36 times in succession, where the standard matrix for R is

$$[R] = \begin{bmatrix} \cos 10^\circ & -\sin 10^\circ \\ \sin 10^\circ & \cos 10^\circ \end{bmatrix},$$

and then perform the calculation.

>> _____

Answer: _____

7. (4 marks) Consider the matrix A and the vector \mathbf{b} given by

$$A = \begin{bmatrix} 2 & 1 & 2 & 3 & 1 & 3 \\ 1 & 2 & 3 & 1 & 3 & 2 \\ 2 & 3 & 1 & 3 & 2 & 1 \\ 3 & 1 & 3 & 2 & 1 & 2 \\ 1 & 3 & 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 & 3 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 3 \\ -6 \\ 7 \\ 3 \\ 0 \\ -7 \end{bmatrix}.$$

- (a) Using MATLAB, find the determinant of A .

Answer: _____

Is A invertible?

Answer: _____

Briefly explain: _____

- (b) After defining A and \mathbf{b} in MATLAB, write the MATLAB code to solve the system $A\mathbf{x} = \mathbf{b}$ for \mathbf{x} , and then perform the calculation.

>> _____

Answer: _____

- (c) Using MATLAB, find the eigenvalues of A . List them in increasing order.

Answer: _____

- (d) Using the MATLAB command `[P,D]=eig(A)`, find an eigenvector associated with the largest eigenvalue of A and scale it by hand so that its components are integer values.

Answer: _____