CAMOSUN

Name: SOLUTIONS

Mark: $\frac{\quad}{25}$

MATH 251

MATLAB Assignment

1. (6 marks) Consider the vectors $\mathbf{u} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -5 \\ 8 \\ 9 \end{bmatrix}$.

   (a) Write the MATLAB code that creates the column vectors $\mathbf{u}$ and $\mathbf{v}$.

   ```
   >> u = [-1; 4; 3]
   >> V = [-5; 8; 9]
   ```

   (b) Write the MATLAB code that calculates the area of the triangle formed by $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$, and perform the calculation.

   ```
   >> norm (cross (u,v))/2
   ```

   Answer: 9

   (c) Write the MATLAB code that calculates the angle, in degrees, between $\mathbf{u}$ and $\mathbf{v}$. Perform the calculation and round your answer to the nearest degree.

   ```
   >> acosd (dot (u,v)/(norm(u) * norm(v)))
   ```

   Answer: 16°

   (d) Write the MATLAB code that calculates $\mathbf{v} - \text{proj}_{\mathbf{u}}(\mathbf{v})$. Perform the calculation and write your answer using fractions.

   ```
   >> V- dot(u,v)/dot(u,u) * u
   ```

   Answer: $\begin{bmatrix} -33/13 \\ -24/13 \\ 21/13 \end{bmatrix}$

2. (5 marks) Consider the unbalanced chemical equation for the following redox reaction involving the nine elements Cu, S, C, N, K, I, O, H, and Cl:

$$\mathrm{CuSCN} + \mathrm{KIO_3} + \mathrm{HCl} \longrightarrow \mathrm{CuSO_4} + \mathrm{KCl} + \mathrm{HCN} + \mathrm{ICl} + \mathrm{H_2O}$$

To balance the equation one must find positive integers $x_1, x_2, \ldots, x_8$ so that

$$x_1\mathrm{CuSCN} + x_2\mathrm{KIO_3} + x_3\mathrm{HCl} \longrightarrow x_4\mathrm{CuSO_4} + x_5\mathrm{KCl} + x_6\mathrm{HCN} + x_7\mathrm{ICl} + x_8\mathrm{H_2O}$$

has the same number of each atom on both sides.

(a) Set up a homogeneous system of linear equations for the variables $x_1, x_2, \ldots, x_8$ and construct its associated 9×9 augmented matrix.

Cu: $x_1 = x_4$

S: $x_1 = x_4$

C: $x_1 = x_6$

N: $x_1 = x_6$

K: $x_2 = x_5$

I: $x_2 = x_7$

O: $3x_2 = 4x_4 + x_8$

H: $x_3 = x_6 + 2x_8$

Cl: $x_3 = x_5 + x_7$

$\Rightarrow$

$$\begin{aligned}
x_1 - x_4 &= 0\\
x_1 - x_4 &= 0\\
x_1 - x_6 &= 0\\
x_1 - x_6 &= 0\\
x_2 - x_5 &= 0\\
x_2 - x_7 &= 0\\
3x_2 - 4x_4 - x_8 &= 0\\
x_3 - x_6 - 2x_8 &= 0\\
x_3 - x_5 - x_7 &= 0
\end{aligned}$$

$$\left[\begin{array}{cccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0\\
0 & 3 & 0 & -4 & 0 & 0 & 0 & -1 & 0\\
0 & 0 & 1 & 0 & 0 & -1 & 0 & -2 & 0\\
0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0
\end{array}\right]$$

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(b) Using MATLAB, find the RREF of the augmented matrix in part (a). Write the result below using fractions, and then, by hand, use the RREF to solve the system of equations and balance the chemical equation.

RREF is

$$\left[\begin{array}{cccccccc|c}
1 & 0 & 0 & 0 & 0 & 0 & 0 & -4/5 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & -7/5 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & -14/5 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -4/5 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -7/5 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -4/5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -7/5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]$$

Let $x_8 = t$ (free variable).

Then $x_1 = \frac{4}{5}t$, $x_2 = \frac{7}{5}t$, $x_3 = \frac{14}{5}t$, $x_4 = \frac{4}{5}t$,

$x_5 = \frac{7}{5}t$, $x_6 = \frac{4}{5}t$, $x_7 = \frac{7}{5}t$, $x_8 = t$

For the smallest positive integer solution, let $t = 5$.

Then $x_1 = 4$, $x_2 = 7$, $x_3 = 14$, $x_4 = 4$, $x_5 = 7$, $x_6 = 4$, $x_7 = 7$, $x_8 = 5$

Answer: $\underline{4}\,CuSCN + \underline{7}\,KIO_3 + \underline{14}\,HCl \longrightarrow \underline{4}\,CuSO_4 + \underline{7}\,KCl + \underline{4}\,HCN + \underline{7}\,ICl + \underline{5}\,H_2O$

3. (3 marks) Consider the matrix A and the vector $\mathbf{b}$ given by

$$A = \begin{bmatrix} 1 & 5 \\ 1 & 3 \\ 1 & -6 \\ 1 & 1 \\ 1 & -5 \\ 1 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 15 \\ 19 \\ 4 \\ 15 \\ 9 \\ 22 \end{bmatrix}.$$

(a) Write the MATLAB code that creates A and $\mathbf{b}$.

```
>> A = [1 5; 1 3; 1 -6; 1 1; 1 -5; 1 2]
>> b = [15; 19; 4; 15; 9; 22]
```

(b) Write the MATLAB code that calculates $(A^T A)^{-1} A^T \mathbf{b}$. Perform the calculation and write your answer using fractions.

```
>> inv(A' * A) * A' * b
```

Answer: $\begin{bmatrix} 14 \\ 61/50 \end{bmatrix}$

4. (2 marks) Write the MATLAB code to evaluate the following complex number, and then perform the calculation, rounding the real and imaginary parts of your answer to two decimal places.

$$\frac{(0.12 + 0.98i)^{100}}{(0.92 - 0.13i)^{40}}$$

```
>> (0.12+0.98i)^100/(0.92-0.13i)^40
```

Answer: 5.06 − 1.49i

5. (3 marks) Consider the matrix A and the vectors $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$ given by

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \\ 26 & 27 & 28 & 29 & 30 \end{bmatrix} \quad \text{and} \quad \mathbf{x}_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \ \mathbf{x}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \ \mathbf{x}_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}.$$

(a) Using MATLAB, find rank(A), and then by hand calculate nullity(A).

Answer: rank(A)= 2, nullity(A)= 3 ← #cols − rank(A) = 5 − 2 = 3

(b) Using MATLAB, evaluate $A\mathbf{x}_1$, $A\mathbf{x}_2$ and $A\mathbf{x}_3$.

Answer: $A\mathbf{x}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $A\mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $A\mathbf{x}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Which of the vectors $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ (if any) are *not* in null(A)?

Answer: $\vec{x}_2$

Briefly explain: $\vec{x}_2 \notin \text{null}(A)$ since $A\vec{x}_2 \neq \vec{0}$

6. (2 marks) Write the MATLAB code that transforms the vector $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$ by applying the linear transformation R to it a total of 36 times in succession, where the standard matrix for R is

$$[R] = \begin{bmatrix} \cos 10^\circ & -\sin 10^\circ \\ \sin 10^\circ & \cos 10^\circ \end{bmatrix},$$

and then perform the calculation.

```
>> [cosd(10) -sind(10); sind(10) cosd(10)]^36 * [4; 5]
```

Answer: $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Note: R rotates $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$ 10° counterclockwise. Applying R 36 times will rotate the vector $36 \times 10^\circ = 360^\circ$ counterclockwise, i.e. one full rotation around a circle. It's not surpringing, therefore, that the result is the same vector $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$. Here $[R]^{36} = I$.

7. (4 marks) Consider the matrix A and the vector $\mathbf{b}$ given by

$$A = \begin{bmatrix} 2 & 1 & 2 & 3 & 1 & 3 \\ 1 & 2 & 3 & 1 & 3 & 2 \\ 2 & 3 & 1 & 3 & 2 & 1 \\ 3 & 1 & 3 & 2 & 1 & 2 \\ 1 & 3 & 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 & 3 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 3 \\ -6 \\ 7 \\ 3 \\ 0 \\ -7 \end{bmatrix}.$$

(a) Using MATLAB, find the determinant of A.

Answer: -216

Is A invertible?

Answer: Yes

Briefly explain: A is invertible since $\det(A) \neq 0$.

(b) After defining A and $\mathbf{b}$ in MATLAB, write the MATLAB code to solve the system $A\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$, and then perform the calculation.

\>> A\b or inv(A)*b

Answer: $\begin{bmatrix} -1 \\ 4 \\ 2 \\ 3 \\ -6 \\ -2 \end{bmatrix}$

(c) Using MATLAB, find the eigenvalues of A. List them in increasing order.

Answer: $-3, -2, -1, 1, 3, 12$.

(d) Using the MATLAB command `[P,D]=eig(A)`, find an eigenvector associated with the largest eigenvalue of A and scale it by hand so that its components are integer values.

Answer: $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

The last diagonal entry of D is 12, which is the largest eigenvalue. The values in the last column of P are all the same, about 4.0825e-01, or 0.40825 (actual value is $\frac{1}{\sqrt{6}}$). Dividing through the last column of P by this value to scale the eigenvector gives an eigenvector whose components are all 1.