

# Diagonalization

**Definition:** An  $n \times n$  matrix  $A$  is said to be **diagonalizable** if there is a diagonal matrix  $D$  and an invertible  $n \times n$  matrix  $P$  such that  $P^{-1}AP = D$  (or equivalently,  $AP = PD$  or  $A = PDP^{-1}$ ).

**Theorem:** Let  $A$  be an  $n \times n$  matrix. Then  $A$  is diagonalizable if and only if  $A$  has  $n$  linearly independent eigenvectors.

More precisely, there exist an invertible matrix  $P$  and a diagonal matrix  $D$  such that  $P^{-1}AP = D$  if and only if the columns of  $P$  are  $n$  linearly independent eigenvectors of  $A$  and the diagonal entries of  $D$  are the eigenvalues of  $A$  corresponding to the eigenvectors in  $P$  in the same order.