CAMOSUN

Name: ______________________________

Mark: $\overline{}$ **25**

MATH 251
Assignment 7

1. (4 marks) Suppose

$$A = \begin{bmatrix} 1 & -2 & 3 & 4 \\ 2 & -4 & 2 & 4 \\ -1 & 2 & 1 & 0 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and $W = \text{col}(A)$ is the column space of A.

(a) Find an orthogonal basis for W.

(b) Find $\text{proj}_W(\mathbf{v})$ and $\text{perp}_W(\mathbf{v})$, where $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

2. (5 marks) Find a QR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

3. (4 marks) Orthogonally diagonalize the following matrix, whose characteristic polynomial is $-(\lambda - 2)(\lambda - 8)^2$, by finding an orthogonal matrix Q and a diagonal matrix D such that $Q^T AQ = D$.

$$A = \begin{bmatrix} 5 & 0 & 3 \\ 0 & 8 & 0 \\ 3 & 0 & 5 \end{bmatrix}$$

4. (5 marks) Suppose A is a 3×3 symmetric matrix having eigenvalues $\lambda_1 = -9$ and $\lambda_2 = 9$ and corresponding eigenspaces $E_{\lambda_1} = \text{span}\left(\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}\right)$ and $E_{\lambda_2} = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}\right)$.

   (a) Orthogonally diagonalize A by finding an orthogonal matrix Q and a diagonal matrix D such that $Q^T AQ = D$.

   (b) Find A.

5. (3 marks) Find the least squares solution of the following inconsistent linear system.

$$\begin{cases} x + y - 2z = 21 \\ x + 2y + z = 7 \\ 2x - y + z = 98 \\ x - y - z = 91 \end{cases}$$

6. (4 marks) Find the least squares approximating line for the points

$$(-4, 3), (-2, -1), (-1, -2), (0, 1), (2, 4)$$

and compute the corresponding least squares error, rounded to two decimal places: