CAMOSUN

Name: ______________________

Mark: $\overline{}$ **25**

MATH 251
Assignment 6

1. (9 marks) Consider the matrix

$$A = \begin{bmatrix} -3 & -4 & 4 \\ 4 & 5 & -4 \\ 2 & 2 & -1 \end{bmatrix}.$$

   (a) Find the eigenvalues of A.

   (b) Is A invertible? Briefly explain.

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(c) Find the eigenspaces corresponding to each eigenvalue of A.

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(d) Is A diagonalizable? Briefly explain. If A is diagonalizable, then find an invertible matrix P and a diagonal matrix D so that $P^{-1}AP = D$.

(e) Evaluate A^{100}.

2. (4 marks) Find the eigenvalues and corresponding eigenspaces of $A = \begin{bmatrix} 3 & -5 \\ 1 & 7 \end{bmatrix}$.

3. (2 marks) For what value(s) of k (if any) is the set

$$\mathcal{B} = \left\{ \begin{bmatrix} k \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -k \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ k \\ 2 \end{bmatrix} \right\}$$

an orthogonal basis of $\mathbb{R}^3$?

4. (2 marks) Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \quad \text{and} \quad \mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix},$$

and note that $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ forms an orthogonal basis of $\mathbb{R}^4$. Express $\mathbf{u}$ as a linear combination of these basis vectors. Give the coordinate vector $[\mathbf{u}]_\mathcal{B}$ of $\mathbf{u}$ with respect to $\mathcal{B}$.

5. (1 mark) Find the inverse of the following orthogonal matrix.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

6. (3 marks) Find a basis for $W^\perp$ if $W = \text{span}(\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3)$ and

$$\mathbf{w}_1 = \begin{bmatrix} 1 \\ 3 \\ -9 \\ 6 \end{bmatrix}, \qquad \mathbf{w}_2 = \begin{bmatrix} -1 \\ -2 \\ 6 \\ -4 \end{bmatrix}, \qquad \mathbf{w}_3 = \begin{bmatrix} 2 \\ 7 \\ -21 \\ 14 \end{bmatrix}.$$

7. (4 marks) Consider the vectors

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \qquad \mathbf{u}_2 = \begin{bmatrix} -2 \\ 2 \\ 3 \\ 2 \end{bmatrix}, \qquad \mathbf{u}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \qquad \text{and} \qquad \mathbf{v} = \begin{bmatrix} -7 \\ -8 \\ 7 \\ 1 \end{bmatrix}.$$

Suppose $\mathcal{B} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a basis for a subspace W of $\mathbb{R}^4$. Note that $\mathcal{B}$ is an orthogonal set.

(a) Find the orthogonal projection of $\mathbf{v}$ onto W.

(b) Find the orthogonal decomposition of $\mathbf{v}$ with respect to W; in other words, find vectors $\mathbf{a}$ in W and $\mathbf{b}$ in $W^\perp$ such that $\mathbf{v} = \mathbf{a} + \mathbf{b}$.

(c) Find an orthonormal basis for W.