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## Mark:

 25
## MATH 251 <br> Assignment 5

1. (1 mark) Write $-3 i$ in phasor form using degrees and then evaluate the product $(-3 i)\left(5 / 25^{\circ}\right)$. Express your answer in phasor form.
2. ( 3 marks) Evaluate $(1-i \sqrt{3})^{14}$. Express your answer in both Euler and rectangular form, using exact values in both cases.
3. (4 marks) Find all the cube roots of $44+117 i$. Express each root in the rectangular form $a+b i$ with $a$ and $b$ rounded to three decimal places. Plot each of the roots in the complex plane.

4. (2 marks) Suppose $z_{1}=a_{1}+b_{1} i$ and $z_{2}=a_{2}+b_{2} i$. Express the quotient $z=z_{1} / z_{2}$ in the form $a+b i$.
5. (2 marks) Show that $\mathbf{v}=\left[\begin{array}{r}-3 \\ 0 \\ 1\end{array}\right]$ is an eigenvector of $A=\left[\begin{array}{rrr}4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8\end{array}\right]$ and find the corresponding eigenvalue.
6. (4 marks) Suppose $\lambda_{1}=3$ is an eigenvalue of $A=\left[\begin{array}{rr}4 & 5 \\ -2 & k\end{array}\right]$.
(a) Find $k$.
(b) Find the other eigenvalue, $\lambda_{2}$.
(c) Find one eigenvector associated with each eigenvalue $\lambda_{1}$ and $\lambda_{2}$.
7. (3 marks) Use Cramer's Rule to solve the following system.

$$
\left\{\begin{aligned}
x_{1}-3 x_{2}+x_{3} & =4 \\
2 x_{1}-x_{2} & =-2 \\
4 x_{1}-3 x_{3} & =0
\end{aligned}\right.
$$

8. (4 marks) Consider the matrix

$$
A=\left[\begin{array}{ccc}
1 & 0 & \tan \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{array}\right]
$$

(a) Find and simplify $\operatorname{det}(A)$.
(b) Use the cofactor method to find $A^{-1}$.
9. (2 marks) Use determinants to find the volume of the parallelepiped formed by the vectors

$$
\mathbf{v}_{1}=\left[\begin{array}{r}
1 \\
-2 \\
3
\end{array}\right], \quad \mathbf{v}_{2}=\left[\begin{array}{r}
5 \\
1 \\
-7
\end{array}\right], \quad \mathbf{v}_{3}=\left[\begin{array}{r}
1 \\
-6 \\
0
\end{array}\right] .
$$

