CAMOSUN

Name: ____________________

Mark: $\overline{}$ **25**

MATH 251

Assignment 4

1. (3 marks) Find an LU factorization of $A = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ -6 & -4 & 0 \end{bmatrix}$.

2. (3 marks) Solve the following system using the LU method.

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 7 & 2 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

3. (2 marks) Let S be the set of vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ in $\mathbb{R}^2$ satisfying the property $y = \pm x$. Either prove that S forms a subspace of $\mathbb{R}^2$ or give a counterexample to show that it does not.

4. (3 marks) Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -2 & -5 & -7 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -4 \\ 3 \\ -1 \end{bmatrix}$. Determine whether or not $\mathbf{v}$ belongs to the following subspaces of $\mathbb{R}^3$.

   (a) $\operatorname{col}(A)$

   (b) $\operatorname{null}(A)$

5. (5 marks) Suppose

$$A = \begin{bmatrix} 1 & -3 & 2 & 1 \\ -2 & 6 & -5 & 1 \\ 4 & -12 & 7 & 7 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -3 & 0 & 7 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -2 & 4 \\ -3 & 6 & -12 \\ 2 & -5 & 7 \\ 1 & 1 & 7 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the column space of A consisting of columns of A.

(b) Express each column vector of A that is not in your basis from part (a) as a linear combination of basis vectors.

(c) Find a basis for the row space of A consisting of rows of A.

(d) Find a basis for the null space of A.

(e) Find rank(A) and nullity(A).

6. (3 marks) Let $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and suppose T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ satisfying $T(\mathbf{i}) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $T(\mathbf{j}) = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$.

   (a) Find $T(\mathbf{v})$.

   (b) Find $T^{-1}(\mathbf{v})$.

7. (3 marks) Let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$.

   (a) Find the coordinate vector $[\mathbf{w}]_{\mathcal{B}}$ of $\mathbf{w}$ with respect to the basis $\mathcal{B} = \{\mathbf{u}, \mathbf{v}\}$ of $\mathbb{R}^2$.

   (b) If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation and

$$T(\mathbf{u}) = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \quad \text{and} \quad T(\mathbf{v}) = \begin{bmatrix} 4 \\ 8 \\ -1 \end{bmatrix},$$

   then find $T(\mathbf{w})$.

8. (3 marks) Consider the linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$ that reflects a vector about the line $y = -x$, then rotates it clockwise by 30°, and then finally projects it onto the y-axis.

   (a) Find the standard matrix of T.

   (b) Use your answer from part (a) to find $T(\mathbf{v})$, where $\mathbf{v} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$. Give an exact answer.