CAMOSUN

Name: ______________________________

Mark: $\overline{}$ **25**

MATH 251
Assignment 3

1. (2 marks) Suppose A is a 2×4 matrix and B is a 3×2 matrix. Find the size of matrix C in each equation, assuming the matrix operations are well-defined.
   (a) $ACB = I_2$
   (b) $4A^T - 3C = O$

2. (2 marks) Let $A = \begin{bmatrix} 1 & a \\ 2 & 1 \end{bmatrix}$. If $A^2 = 2A$, then find a.

3. (3 marks) Evaluate (if possible) $B^T B + 4A^2 - 3I_2$, where $A = \begin{bmatrix} 7 & 1 \\ -5 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 0 \\ 1 & 4 \\ 9 & -1 \end{bmatrix}$.

4. (3 marks) Write D as a linear combination of A, B, and C (if possible), where

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}, \quad \text{and} \quad D = \begin{bmatrix} -4 & -8 \\ -3 & 9 \end{bmatrix}.$$

5. (3 marks) Determine whether AB is in $\operatorname{span}(A, B)$, where $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 4 \\ 0 & 3 \end{bmatrix}$.

6. (3 marks) If $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & 5 \end{bmatrix}$, then solve the following equation for X.

$$XAA^T + B - 3I_2 = O$$

7. (2 marks) Let $A = \begin{bmatrix} 1 & 4 & 0 \\ -2 & 2 & -3 \\ 5 & 1 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 1 & 7 \\ -2 & 2 & -3 \\ 1 & 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 10 & -3 \\ 5 & 1 & 7 \end{bmatrix}$.

(a) Find an elementary matrix E satisfying the equation $EA = B$.

(b) Find an elementary matrix E satisfying the equation $EA = C$.

8. (1 mark) Find the inverse of $A = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$.

9. (3 marks) Let $A = \begin{bmatrix} 1 & 0 \\ -4 & 3 \end{bmatrix}$. Write A^{-1} and A each as a product of two elementary matrices.

10. (3 marks) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix}$.

   (a) Find A^{-1} using the Gauss-Jordan method for computing the inverse.

   (b) Use A^{-1} to solve the system $A\mathbf{x} = \mathbf{b}$.