



Name: \_\_\_\_\_

Mark: 25

## MATH 251

### Assignment 2

1. (3 marks) Consider the following system of linear equations.

$$\begin{cases} 6x_1 - 3x_2 + 2x_4 = 7 \\ x_2 + 2x_3 + 5x_4 = 15 \\ 3x_3 - x_4 = -5 \\ 3x_4 = 6 \end{cases}$$

- (a) Construct the augmented matrix for this system. Is it in row echelon form (REF)? Is it in reduced row echelon form (RREF)?
- (b) Solve the system by using back substitution.

2. (3 marks) Solve the system of equations using the Gauss-Jordan method.

$$\begin{cases} x - 2y + z = 0 \\ 2x - 2y - 6z = 8 \\ -4x + 5y + 9z = -9 \end{cases}$$

3. (2 marks) For what value(s) of  $k$  (if any) is the system of linear equations having the following augmented matrix inconsistent?

$$\left[ \begin{array}{cc|c} 1 & k & 2 \\ k & 4 & 4 \end{array} \right]$$

4. (5 marks) Consider the two planes  $x + 3y - 2z = 2$  and  $3x - 5y + 8z = -8$ .
- (a) Find parametric equations for the line of intersection of these two planes.
  - (b) Determine whether the line found in part (a) and the line described by

$$x = 6 + 3t, \quad y = -9 - 4t, \quad z = 5 + t$$

intersect and, if they do, find their point of intersection.

5. (3 marks) Consider the vectors

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}.$$

- (a) Determine whether  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are linearly independent or linearly dependent. If they are linearly dependent, find a dependent relationship among the vectors.
- (b) Determine whether or not  $\mathbf{v}$  is in  $\text{span}(\mathbf{u}, \mathbf{w})$ .

6. (5 marks) A supermarket sells three different bags of assorted fruit, each containing a mixture of bananas, apples and oranges: *Awesome Fruits*<sup>™</sup>, *Bodacious Fruits*<sup>™</sup> and *Comfort Fruits*<sup>™</sup>. The number of each fruit in each bag is summarized in the following table.

	Awesome Fruits (AF)	Bodacious Fruits (BF)	Comfort Fruits (CF)
bananas	2	6	2
apples	3	10	4
oranges	2	5	1

Can a customer buy bags of fruit from the supermarket so as to get exactly 16 bananas, 25 apples and 15 oranges? If so, how many bags of each of the three varieties must he or she buy? If there is more than one possibility, then find all the solutions. If it's not possible, then explain why. To support your answer, set up and solve a system of linear equations.

7. (4 marks) Consider the unbalanced chemical equation for the following redox reaction involving the nine elements Cu, S, C, N, K, I, O, H, and Cl:



To balance the equation one must find positive integers  $x_1, x_2, \dots, x_8$  so that



has the same number of each atom on both sides.

- Set up a homogeneous system of linear equations for the variables  $x_1, x_2, \dots, x_8$ .
- Construct the augmented matrix associated with the system of equations in part (a).

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- (c) Find the RREF of the augmented matrix in part (b) by using the `rref` command in Octave or MATLAB.
- (d) Use the RREF from part (c) to solve the system of equations and then balance the chemical equation.