

Name: _

Mark: $\mathbf{25}$

MATH 251 Assignment 1

1. (4 marks) Consider two points A and B in \mathbb{R}^3 . Suppose A = (3, 0, 2) and $\overrightarrow{AB} = \begin{bmatrix} 2\\ -3\\ 6 \end{bmatrix}$.

- (a) Find the coordinates of point B.
- (b) Find two distinct unit vectors that are parallel to \overrightarrow{AB} .
- (c) Find parametric equations for the line passing through points A and B.
- (d) At what point does the line from part (c) intersect the xy-plane?

2. (7 marks) Consider the three points

$$A = (2, 0, -5),$$
 $B = (8, 2, -9),$ $C = (7, 9, -7).$

- (a) Find the area of triangle $\triangle ABC$. Give an exact, simplified answer.
- (b) Find the angle $0^{\circ} \le \theta \le 180^{\circ}$ between \overrightarrow{AB} and \overrightarrow{AC} . Round your answer to two decimal places.
- (c) Determine whether or not $\triangle ABC$ is a right triangle.
- (d) Find the equation, in general form, of the plane passing through the points A, B, and C.

3. (3 marks) Using projections, find the distance between the parallel planes x + y - 2z = 2 and x + y - 2z = 4. Given an exact, simplified answer.

4. (3 marks) Let $\mathbf{u} = [-4, 0, 3]$ and $\mathbf{v} = [2, 5, 1]$. Find vectors \mathbf{p} and \mathbf{q} so that $\mathbf{v} = \mathbf{p} + \mathbf{q}$, \mathbf{p} is parallel to \mathbf{u} , and \mathbf{q} is orthogonal to \mathbf{u} . [Hint: Use projections to find one of the vectors.]

5. (3 marks) Find an equation, in parametric form, of the line passing through the point P = (1, 2, -1) and orthogonal to the plane defined by

$$\begin{cases} x = 6 + 2s - t \\ y = 1 - 3s + 5t \\ z = -7 + s - t. \end{cases}$$

6. (3 marks) Find the vector form of the equation of the plane 3x - 4y + 2z = 12.

7. (2 marks) Find the value(s) of k such that the vector

$$\mathbf{v} = \begin{bmatrix} -1\\k\\2 \end{bmatrix}$$

is orthogonal to the plane 2x + 3y - 4z = 0.