MATH 226 (Winter, 2025)
Test 2

1. (5 marks) Use either the method of undetermined coefficients or annihilators to solve the differential equation $y'' - 6y' + 5y = 8e^x$.

2. (6 marks) Use variation of parameters to find the general solution of $y'' + y = \sec x \tan x$, starting from

$$\begin{cases} u_1' y_1 + u_2' y_2 = 0 \\ u_1' y_1' + u_2' y_2' = f(x) \end{cases}.$$

3. (4 marks) A $5\,\text{kg}$ mass is attached to a spring, causing it to stretch $2\,\text{m}$. The mass is released $1\,\text{m}$ below the equilibrium position with an initial upward velocity of $3\,\text{m/s}$ and is subject to a damping force that is twice its velocity. Set up, **but do not solve**, the initial-value problem (both differential equation and initial conditions) for the equation of motion $x(t)$ of the mass. Recall $g = 9.8\,\text{m/s}^2$.

4. (5 marks) Find a power series solution of the differential equation $\dfrac{dy}{dx} + xy = 0$ and then convert it to a closed form solution.

5. (5 marks) Use Laplace transforms to solve the initial-value problem.

$$y'' + 2y' + y = 1 + \delta(t - 3), \ y(0) = 0, \ y'(0) = 0$$