CAMOSUN

Name: _____

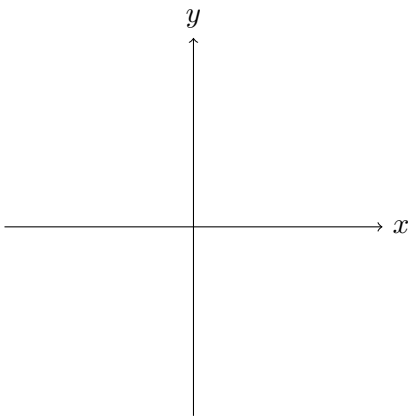
Mark: ____ / 25

MATH 226 (Winter, 2025)

Test 1

1. Consider the first-order differential equation $\dfrac{dy}{dx} = 9x^{1/2}y^{1/3}$.

   (a) (2 marks) Determine and sketch the region in the xy-plane where the differential equation would have a unique solution through (x_0, y_0).



   (b) (3 marks) Show that $y = 8x^{9/4}$ is a solution of the differential equation with initial condition $y(0) = 0$. Could there be other solutions? Briefly explain.

2. (2 marks) State the order of each differential equation and whether it is linear or nonlinear.

(a) $x\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$

(b) $(\tan t)\frac{d^3u}{dt^3} + (\cos t)\frac{du}{dt} + te^t u = \sin t$

(c) $x\left(x + \frac{1}{y}\right)dx + \frac{1}{y}dy = 0$

(d) $\frac{d^2y}{dx^2} - y\frac{dy}{dx} + y = 0$

3. (3 marks) Solve the initial value problem $\frac{dy}{dx} = \sin x \csc y$, $y(0) = \pi/2$. Express y explicitly as a function of x.

4. Solve the following first-order differential equations.

   (a) (3 marks) $\dfrac{dy}{dx} = \dfrac{x^2 + xy + y^2}{x^2}$ [Express y explicitly as a function of x.]

   (b) (3 marks) $(2xy + x + ye^{xy})dx + (x^2 + y + xe^{xy})dy = 0$

5. (3 marks) Given that $y_1 = x$ is a solution of $x^2y'' + xy' - y = 0$, use the **reduction of order** method to find a second solution.

6. (2 marks) Find the general solution of the differential equation $y^{(4)} + 4y'' = 0$.

7. (4 marks) A large 100 liter punch bowl is kept full by adding 10 liters per hour of a 40% alcohol mix. The guests consume 10 liters per hour of the mixed punch. If there were initially 20 liters of alcohol in the bowl, find the amount $A(t)$ of alcohol in the bowl (in liters) at time t (in hours) by setting up and solving a linear differential equation.