Total Differentials

The total differential of a function $f(x, y)$ of two variables is

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy.$$

The following are important total differentials.

$$d(xy) = ydx + xdy$$

$$d\left(\frac{y}{x}\right) = \frac{xdy - ydx}{x^2} \quad \text{and similarly} \quad d\left(\frac{x}{y}\right) = \frac{ydx - xdy}{y^2}$$

$$\frac{1}{2}d(x^2 + y^2) = xdx + ydy$$

$$d\left(\arctan\left(\frac{y}{x}\right)\right) = \frac{xdy - ydx}{x^2 + y^2}$$

$$\frac{1}{2}d\left(\ln\left(x^2 + y^2\right)\right) = \frac{xdx + ydy}{x^2 + y^2}$$