Applications of First-Order Differential Equations

1. Suppose rabbits were introduced to an island in 2015 and thereafter the rabbit population grew at a rate proportional to the number present. Suppose 700 rabbits were counted on the island in 2022 and three years later the count was 2,700.

   (a) How many rabbits were introduced to the island?

   (b) How many rabbits are there predicted to be on the island in 2028?

   (c) How many years after the introduction of the rabbits is the population expected to reach 40,000 rabbits?

2. If the number of bacteria in a culture grows exponentially and triples in size every 10 hours, how long does it take to double in size?

3. Radioactive isotopes decay at a rate proportional to the amount of material present. The term "half-life" refers to the amount of time it takes for half the atoms in a sample of radioactive material to decay. Plutonium-239 (^{239}Pu) is used as a fuel to power nuclear reactors and in the production of nuclear weapons. Its half-life is 24,110 years.

   (a) How long will it take for a fuel rod of ^{239}Pu to lose 90% of its radioactive material?

   (b) How much radioactive material remains from a 5 kg fuel rod of ^{239}Pu after 10,000 years?

4. Suppose your cup of coffee is too hot to drink at 95°C. After 10 minutes it has only dropped to 90°C in a room where the temperature is kept at 20°C.

   (a) What will the temperature be after 20 minutes?

   (b) How long must you wait until the coffee temperature drops to a comfortable 78°C?

5. A 1,000 L tank initially contains 2 kg of salt. If pure water pours into the tank at 5 L/min and the mixed result pours out of the tank, also at 5 L/min, then how much salt is in the tank at time t?

6. Beer containing 6% alcohol is pumped into a tank that initially contains 300 L of beer at 3% alcohol. The rate at which the 6% beer is pumped in is 4 L/min and the mixed beer is drained out at a rate of 6 L/min.

   (a) How many liters of alcohol are in the tank at time t?

   (b) Find the percentage of alcohol in the tank after 60 minutes.

7. Suppose trout in a pond are limited by food and oxygen according to the logistic equation. If there are initially 50 trout, and after 1 year there are 100 trout, how many trout are expected after 2 years if it is assumed that $a = 2$ in the logistic equation model (i.e. the birth rate is 2 trout per year per trout).

8. Consider the balanced chemical equation $2H_2 + O_2 \longrightarrow 2H_2O$ representing the reaction that converts hydrogen gas and oxygen gas into water. Each mole of H_2, O_2 and H_2O has a mass of 2 g, 32 g and 18 g, respectively. Suppose 100 g of H_2 and 1,000 g of O_2 are mixed and the rate at which water is produced is proportional to the amount of reactants present. If after 1 second, 500 g of H_2O is formed, then how much is formed after 2 seconds?