CAMOSUN

Name: ____________________________

MATH 126 (Winter, 2014)
Term Test 3
by George Ballinger

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Answer the questions in the space provided. This test has 10 questions for a total of 25 marks. Express answers to counting problems as integers and show your calculations.

1. (6 marks) How many strings of six letters from the alphabet $\{A, B, C, \ldots, Z\}$ are there

   (a) if letters **cannot** be repeated?

   (b) that start with one of the vowels A, E, I, O or U if letters **can** be repeated?

   (c) that start with A or end with Z if letters **can** be repeated?

   (d) that have at least one A if letters **can** be repeated?

2. (2 marks) A school baseball coach must choose 9 children from among a group of 7 girls and 20 boys to form a team. If the team must have either 2 or 3 girls on it, then in how many ways can the coach form such a team?

3. (3 marks) How many strings of length 9 are there, using symbols from $\{G, O, D\}$, if

   (a) each string contains exactly 4 G's, 3 O's, and 2 D's?

   (b) each string contains exactly 4 G's, 3 O's, and 2 D's and the 3 O's are consecutive (as in "OOO")?

4. (1 mark) What is the minimum number of students required in a discrete mathematics class to be sure that at least three will receive the same grade if there are 10 possible grades, A+, A, A-, B+, B, B-, C+, C, D, and F?

5. (1 mark) What is the coefficient of $x^{27}y^3$ in the binomial expansion of $(x+y)^{30}$?

6. (3 marks) A bowl contains a large mixture of blue, red, green and yellow jelly beans (at least 15 of each). Jelly beans of the same colour are considered identical. Suppose a handful of jelly beans is taken from the bowl.

   (a) How many different handfuls of 15 jelly beans are possible?

   (b) How many different handfuls of 15 jelly beans are possible having exactly 3 red jelly beans and at least 4 green jelly beans?

7. (2 marks) What is the probability that a five-card poker hand from a standard deck of 52 cards contains exactly one ace? Round your answer to three decimal places.

8. (1 mark) Find a recurrence relation for the number of ways, a_n, of depositing n cents into a vending machine using just nickels (5-cent coins) and dimes (10-cent coins) if the order in which the coins are deposited matters.

$$a_n = \underline{\hspace{6cm}} \text{ for } n \geq 10$$

9. (2 marks) Suppose f satisfies the divide-and-conquer recurrence relation

$$f(n) = 3f(n/2) + 5,$$

whenever $n = 2^k$ for $k \in \mathbb{Z}^+$. If $f(1) = 4$, then find $f(131072)$. Recall

$$f(n) = \begin{cases} f(1) + c\log_b n, & \text{if } a = 1, \\ \left[f(1) + \dfrac{c}{a-1}\right] n^{\log_b a} - \dfrac{c}{a-1}, & \text{if } a > 1. \end{cases}$$

10. (4 marks) Consider the sequence of Fibonacci numbers $f_0, f_1, f_2, f_3, \ldots$. Recall that $f_0 = 0$ and $f_1 = 1$.

(a) State the recurrence relation that defines f_n in terms of previous Fibonacci numbers and list the Fibonacci numbers $f_2, f_3, f_4, f_5, f_6, f_7$ in the space provided.

$f_n =$ ________________ for $n \geq 2$. The sequence begins 0, 1, ___, ___, ___, ___, ___, ___, ...

(b) Use **strong induction** to prove that $f_n \geq n$ for integers $n \geq 5$.