

-
3. (2 marks) Suppose 11 Americans (including Donald Trump) and 17 Canadians (including Doug Ford) have volunteered to participate in a clinical trial studying erectile dysfunction. Suppose 14 of these 28 volunteers are selected to receive an experimental drug while the rest are to get a placebo. In how many ways can the selection of these 14 be made if there must be equal numbers of Americans and Canadians and the selection must include both Donald Trump and Doug Ford?
4. (6 marks) Passengers aboard flight ABC123 can order any one of five meals: steak, chicken, fish, pasta or vegetarian. Suppose each of the passengers orders a meal.
- (a) What is the minimum number of passengers required to guarantee that at least 4 ordered the same meal?
- (b) If there are 25 passengers, then how many different combinations of orders are possible?
- (c) If there are 25 passengers, then how many different combinations of orders are possible if exactly six passengers order fish and at least three order pasta?

5. (2 marks) What is the probability that a five-card poker hand from a standard deck of 52 cards contains exactly three 6's. Round your answer to six decimal places.

6. (1 mark) Evaluate $C(30, 0) + C(30, 1) + C(30, 2) + C(30, 3) + \cdots + C(30, 30)$.

7. (2 marks) The number of comparisons, $f(n)$, required (in the worst case) to sort a list of n elements using the merge sort algorithm can be described recursively by $f(1) = 0$ and

$$f(n) = 2f(n/2) + (n - 1),$$

whenever $n = 2^k$ for $k \in \mathbb{Z}^+$. How many comparisons are required to sort a list of size $n = 16$?

8. (7 marks)

(a) Use **strong induction** to prove that every amount of postage of 12 cents or more can be formed using just 3-cent and 7-cent stamps.

(b) Let a_n represent the number of ways of forming n cents of postage using just 3-cent and 7-cent stamps if the order in which the stamps are used to form n cents matters.

(i) Find a_{13} .

(ii) Find a recurrence relation for a_n .

(iii) Find a_{50} given the following values for a_n .

| | | | | | | | | | | | | | | | |
|-------|----|----|----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|
| n | 12 | 13 | 14 | 15 | 16 | ... | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| a_n | 1 | ? | 1 | 1 | 4 | ... | 122 | 139 | 122 | 173 | 224 | 204 | 249 | 346 | ? |