CAMOSUN

Name: ____________________

MATH 126 (Winter, 2018)
Term Test 2
by George Ballinger

Answer the questions in the space provided.
This test has 11 questions for a total of 30 marks.

1. (1 mark) Given the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$, find $f(A)$, where $A = [-5, 3]$ is a closed interval.

2. (1 mark) Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ that is one-to-one and onto but is not the identity function. You do not have to prove that your function is one-to-one and onto.

3. (1 mark) Which *one* of the following expressions defines an actual function from $\mathbb{R}$ to $\mathbb{R}$? Circle your answer. No justification is required.

   (i) $f(x) = \sqrt{x}$ (iii) $f(x) = \pm x$ (v) $f(x) = \lfloor x \rfloor$

   (ii) $f(x) = 1/x$ (iv) $f(x) = x!$ (vi) $f(x) = \log_2 x$

4. (1 mark) Which *one* of the following sequences $\{a_n\}$ is *not* a solution of the recurrence relation $a_n = 2a_{n-1} + 3a_{n-2}$. Circle your answer. No justification is required.

   (i) $\{0\} = 0, 0, 0, 0, 0, \ldots$ (iii) $\{(-1)^n\} = -1, 1, -1, 1, -1, \ldots$

   (ii) $\{n - 1\} = 0, 1, 2, 3, 4, \ldots$ (iv) $\{3^n\} = 3, 9, 27, 81, 243, \ldots$

5. (2 marks) If the time now is 2pm, then what time of day was it $10,000$ hours ago?

6. (4 marks) Using the definition of onto, prove that if $f : B \to C$ is onto and $g : A \to B$ is onto, then $f \circ g : A \to C$ is onto.

7. Find the greatest common divisor of 630 and 196 by using

   (a) (2 marks) the prime factorization of each number.

   (b) (2 marks) the Euclidean Algorithm.

8. (2 marks) Give an example of a *composite* number n such that the numbers 14, 15 and n are pairwise relatively prime.

9. (2 marks) Prove that the union of two countably infinite sets

$$A = \{a_1, a_2, a_3, a_4, \ldots\} \text{ and } B = \{b_1, b_2, b_3, b_4, \ldots\}$$

is countably infinite. Listing the elements of $A \cup B$ in a sequence will suffice, provided it's clear how the sequence is constructed.

10. Consider four-digit octal (base-8) numbers of the form $n = (abcd)_8$.

    (a) (2 marks) Convert $(7143)_8$ and $(1060)_8$ to decimal (base-10), showing your calculations.

    (b) (3 marks) Prove that if $n = (abcd)_8$, then $n \equiv d \pmod 8$. You may not use any theorems about congruence or divisibility (only their definitions) unless you also prove them.

    (c) (1 mark) Convert $(7143)_8$ to binary (base-2). You do not need to show any work.

11.

(a) (4 marks) Use the Principle of Mathematical Induction to prove that $n! \geq 3(2^n)$ for all integers $n \geq 5$.

(b) (2 marks) Prove that 2^n is $O(n!)$ by finding positive constants (i.e. witnesses) C and k from the definition of big-O.